Freeze-out hypersurface and resonance decay influence on femtoscopic observables

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Outline

- Single freeze-out approximation
  - Theoretical foundations
  - Numerical implementation
  - Freeze-out hypersurface
- Femtoscopy definitions
  - Separation distribution
  - Correlation function
  - Gaussian fit
  - The meaning of "HBT radii"
- Results from Therminator
  - Identical particles
    - Sepration distributions
    - Correlation functions
  - Influence of the strongly-decaying resonances
    - Influence on size
    - Influence on lambda
    - Influence on the shape of the correlation function
  - Non-identical correlations

Single freeze-out approximation analytic version (the Cracow Model)

- Developed by W. Broniowski and W. Florkowski
- Main features and assumptions of the model
  - Matter is thermally equilibrated. The abundances of particles are determined by the temperature, through statistical model. The temperature at which particles are emitted is the same ($T_{chem} = T_{kin} = T$)
  - Complete treatment of resonances (381 particle types)
  - Special choice of freeze-out hypersurface: ($\tau=\sqrt{t^2-x^2-y^2-z^2}$)
  - Only 4 parameters T,μ_B (fixed by abundance ratios), proper time at freeze-out τ and transverse size ρ_{max}.
  - Hubble-like flow: $u^\mu = x_\mu/\tau = t/\tau\ (1,\ x/t,\ y/t,\ z/t)$

Numerical implementation: THERMINATOR

- THERMINATOR starts from a full emission function of primordial particles (no resonance feed-down yet):

$$S(x, p) = \frac{dN}{dp_T \, dy \, d\phi_p \, d\alpha_T \, d\alpha_{\parallel} \, d\phi_s}$$

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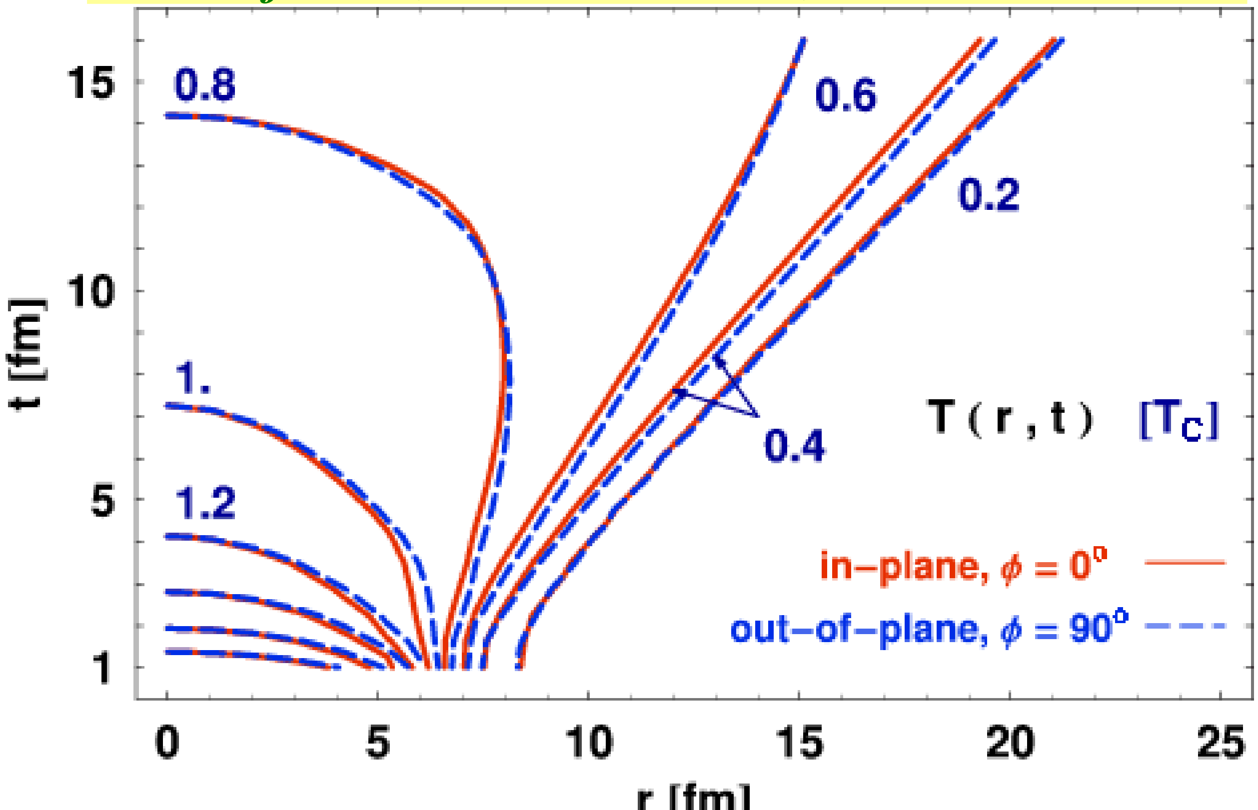
- It performs a Monte-Carlo integration of the emission function in six dimensions and then proceeds to decay all unstable particles, also with a Monte-Carlo method
  - Gives space-time freeze-out points for all particles
  - Emission function based on Blast-wave – has strong flows
  - Relative and absolute multiplicities taken from the chemical model, with parameters fit to the data

nucl-th/0504047 ***http://hirg.if.pw.edu.pl/en/therminator/***

The importance of freeze-out hypersurface

- The Cracow single freeze-out model implemented a particular shape of freeze-out hypersurface where $\tau = const$
- Commonly used Blast-wave models have a hypersurface defined as $t = const$
- Hydrodynamic calculations usually produce a different shape in the t-ρ plane, so "generalized Blast-wave" was used

M. Chojnacki, W. Florkowski, nucl-th/0603035



Different freezeout hypersurface – "BlastWave" with resonances

- In Therminator we have complete freedom of choice of the emission function. We use e.g. "generalized BlastWave":

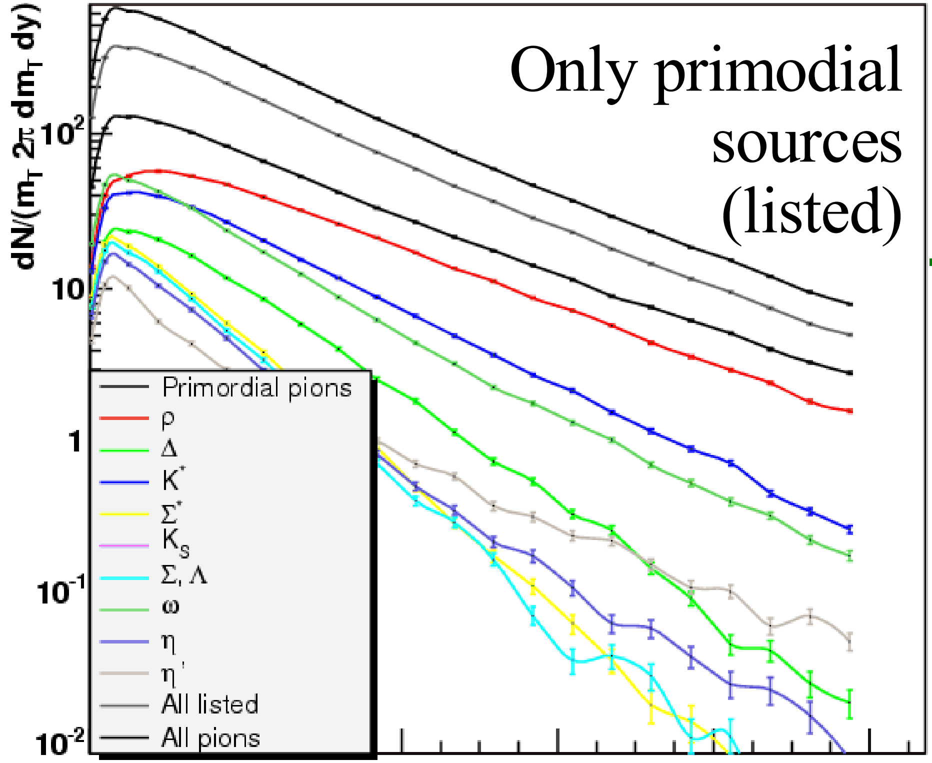
$$\frac{dN}{dp_\perp d\phi_p dy d\rho d\phi_S d\alpha_\parallel} = \frac{p_T \rho}{(2\pi)^3} (\tau + a\rho) \left[m_\perp \cosh\left(\alpha_\parallel - y\right) - a p_\perp \cos\left(\phi_P - \phi_S\right) \right]$$

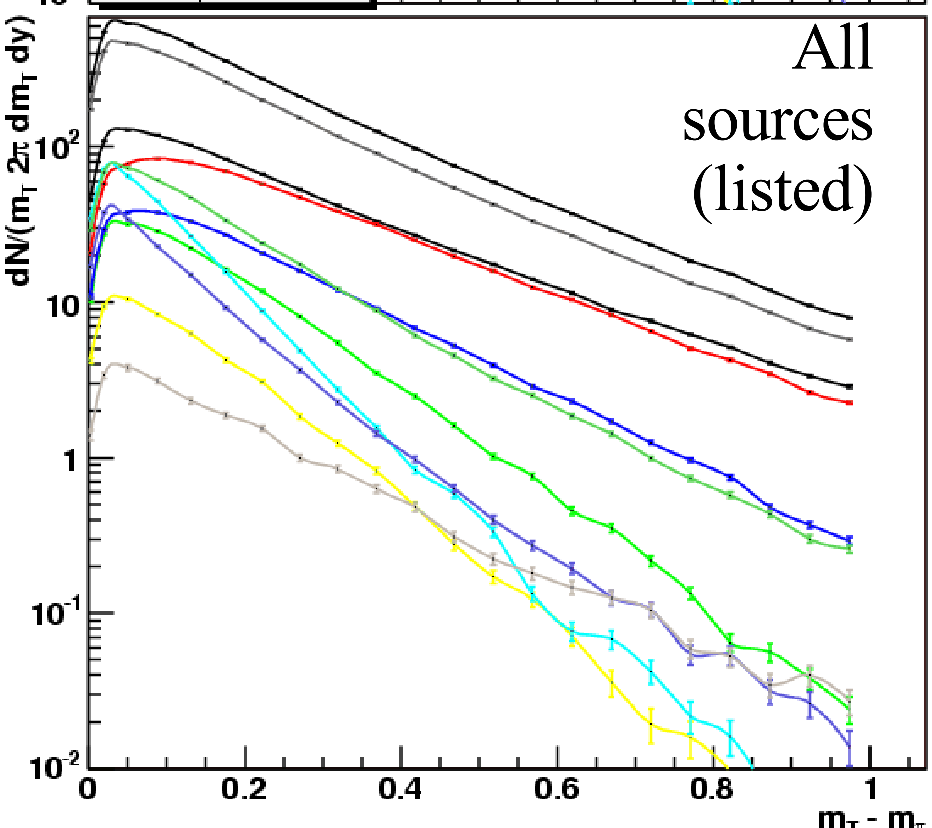
$$\times \left\{ \exp\left[\beta m_\perp \frac{1}{\sqrt{1 - v_\perp^2}} \cosh\left(\alpha_\parallel - y\right) - \beta p_\perp \frac{v_\perp}{\sqrt{1 - v_\perp^2}} \cos\left(\phi_P - \phi_S\right) - \beta\mu \right] \pm 1 \right\}^{-1}$$

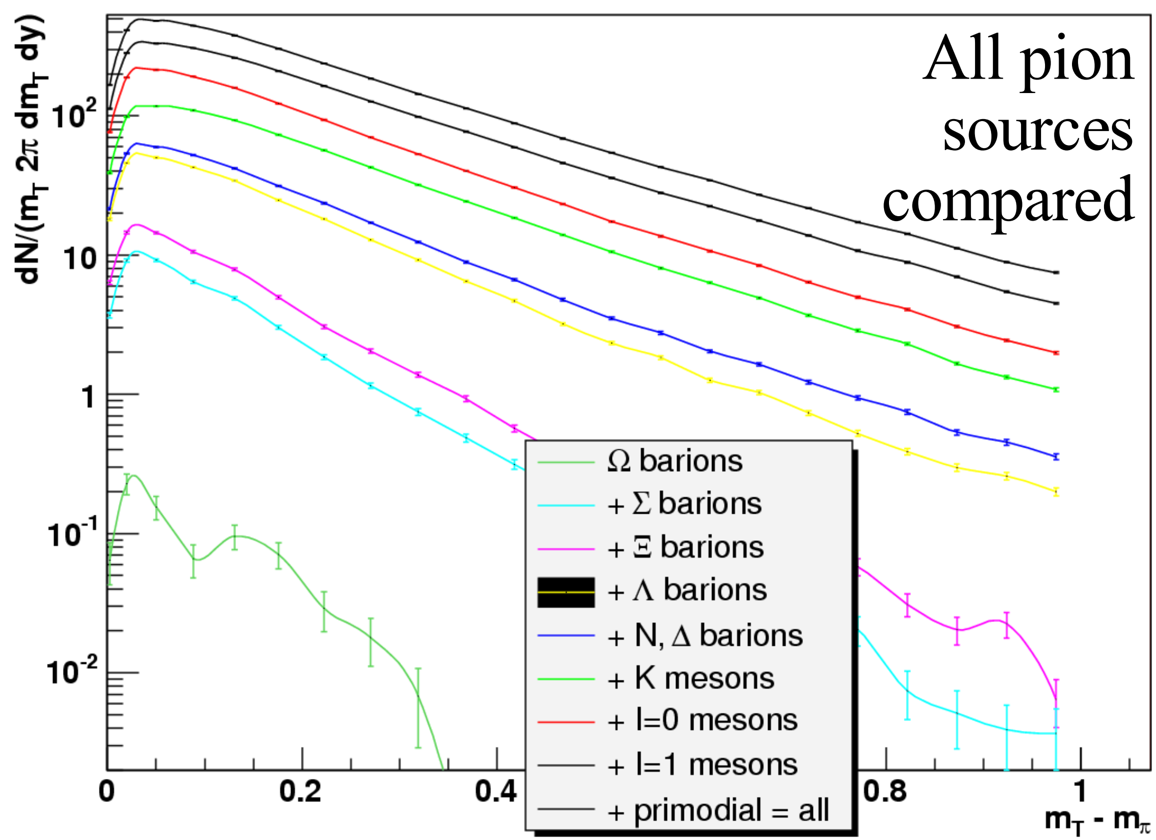
- The a parameter from the $\tau + a\rho$ term determines the shape of the freeze-out in ρ-t plane
- Thermodynamical parameters (T, μ_B) stay the same. ρ_{max} and τ and v_T fitted, so that STAR pion and kaon spectra is reproduced.
- Please note this "Blast-wave" has all resonances included

Resonance influence on (pion) slopes

- Please note that “most important” resonances account for 60%(73%) of all pions (depending on how you count)







Emission function

- We use the "emission function" $S(x,p)$; a probability to emit particle with momentum p from point x.
- We can then define single particle spectra:

$$P_1(\vec{p}) = E\frac{dN}{d^3p} = \int d^4x\, S(x,p)$$

as well as two-particle spectra:

$$P_2(\vec{p}_a, \vec{p}_b) = E_a E_b \frac{dN}{d^3p_a d^3p_b} = \int S(\mathbf{x_1}, \mathbf{x_2}, \mathbf{p_1}, \mathbf{p_2})\, d^4x_1\, d^4x_2$$

- And the correlation function is:

$$C(\vec{q}, \vec{K}) = \frac{P_2^C(\vec{p}_a, \vec{p}_b)\,\delta(\vec{q} - \vec{p}_a + \vec{p}_b)\,\delta(\vec{K} - 1/2(\vec{p}_a + \vec{p}_b))}{P_2(\vec{p}_a, \vec{p}_b)\,\delta(\vec{q} - \vec{p}_a + \vec{p}_b)\,\delta(\vec{K} - 1/2(\vec{p}_a + \vec{p}_b))}$$

Connection to separation distribution

- If we neglect the event-wide correlations and leave only femtoscopical (two-particle) ones, we write:

$$\int S(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{p_1}, \boldsymbol{p_2}) d^4 \boldsymbol{x_1} d^4 \boldsymbol{x_2} \approx \int S_1(\boldsymbol{x_1}, \boldsymbol{p_1}) S_2(\boldsymbol{x_2}, \boldsymbol{p_2}) d^4 \boldsymbol{x_1} d^4 \boldsymbol{x_2}$$

- And we can define the separation distribution

$$S(\boldsymbol{r}^*, p_a, p_b) \stackrel{\text{def}}{=} \int S_1(x_1, p_a) S_2(x_2, p_b) \delta(r^* - \boldsymbol{x_1} + \boldsymbol{x_2}) d^4 x_1 d^4 x_2 = \int S_1(x_1, p_a) S_2(r^* - x_1, p_b) d^4 x_1$$

- In the uncorrelated case we have:

$$P_2(\vec{p}_a, \vec{p}_b) = \int S(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{p_1}, \boldsymbol{p_2}) d^4 x_1 d^4 x_2 = \int S(\boldsymbol{r}^*, p_a, p_b) d^4 r$$

- While in the correlated case we need the separation distribution itself:

$$P_2^C(p_a, p_b) = \int S(\boldsymbol{r}^*, p_a, p_b) \left|\Psi(\boldsymbol{r}^*, \boldsymbol{k}^*)\right|^2 d^4 r$$

The wave function

- For the correlation function we need the wave function
- For identical non-charged particles we have:

$$\left|\Psi(\boldsymbol{r}^*,\boldsymbol{k}^*)\right|^2=1+\cos(\boldsymbol{q}_{inv}\cdot\boldsymbol{r}^*)$$

- If the particles are charged, then:

$$\Psi(\boldsymbol{k}^*,\boldsymbol{r}^*)=e^{i\delta_c}\sqrt{A_c(\eta)}\frac{1}{\sqrt{2}}\left[e^{-i\boldsymbol{k}^*\boldsymbol{r}^*}F(-i\eta,1,i\xi^+)\pm(-1)^S e^{i\boldsymbol{k}^*\boldsymbol{r}^*}F(-i\eta,1,i\xi^-)\right]$$

- If they are nonidentical, the correlation function can also be calculated. We have then:

$$\Psi(\boldsymbol{k}^*,\boldsymbol{r}^*)=e^{i\delta_c}\sqrt{A_c(\eta)}\left[e^{-i\boldsymbol{k}^*\boldsymbol{r}^*}F(-i\eta,1,i\xi)\right]$$

where $\xi^{+/-}=\boldsymbol{k}^*\boldsymbol{r}^*\pm k^*r^*\equiv\rho(1\pm\cos(\theta^*)),\ \rho=k^*r^*,\ \eta=(k^*a)^{-1},\ a=(\mu z_1 z_2 e^2)^{-1}$

$A_c(\eta)$ is the Gamow factor, and F is the confluent hypergeometric function.

Numerical methods

- All analytic methods can be naturally transformed into numerical ones by performing "hadronization" using a modification of Monte-Carlo integration of the Cooper-Frye formula. Generating sets of random numbers *(x,p)* distributed according to the probability density *S(x,p)*, we can interpret each of them as a particle and combine them in pairs.

- Then, by the definition of the Monte-Carlo procedure, each integral of the emission function can be turned into a simple pair counting (with or without weight). E.g.

$$\int d^4 r\, S(\boldsymbol{r}^*, \boldsymbol{p}_1, \boldsymbol{p}_2) \stackrel{\text{def}}{=} \sum_i \sum_{j\neq i} \delta(\boldsymbol{p}_i - \boldsymbol{p}_a)\, \delta(\boldsymbol{p}_j - \boldsymbol{p}_b),$$

$$\int d^4 r\, S(\boldsymbol{r}_*, \boldsymbol{p}_1, \boldsymbol{p}_2) \left|\Psi\right|^2 \stackrel{\text{def}}{=} \sum_i \sum_{j\neq i} \delta(\boldsymbol{p}_i - \boldsymbol{p}_a)\, \delta(\boldsymbol{p}_j - \boldsymbol{p}_b) \left|\Psi\right|^2$$

- This is the procedure used in our studies

Radius extraction in data

- In model studies the known fit form for quantum-statistics only correlation function is used, giving "HBT radii":

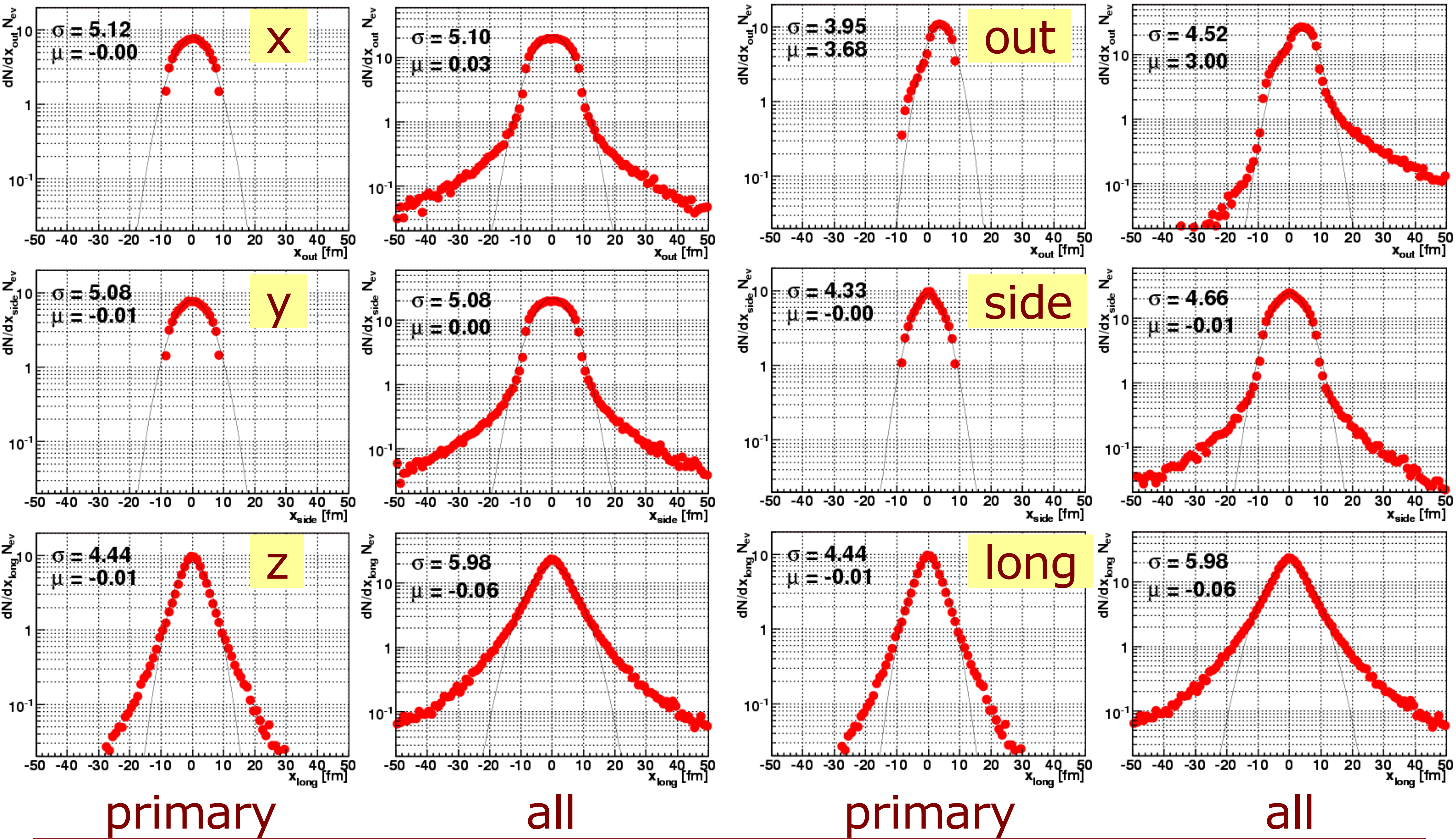
$$C(\vec{q}) = 1 + \lambda \exp(-R_{out}^2 q_{out}^2 - R_{side}^2 q_{side}^2 - R_{long}^2 q_{long}^2)$$

- To fit the experimental data or the QS+Coulomb model function, an approximation is used stating that Coulomb and symmetrization of the wave function factorize, which gives "Bowler-Sinyukov" formula:

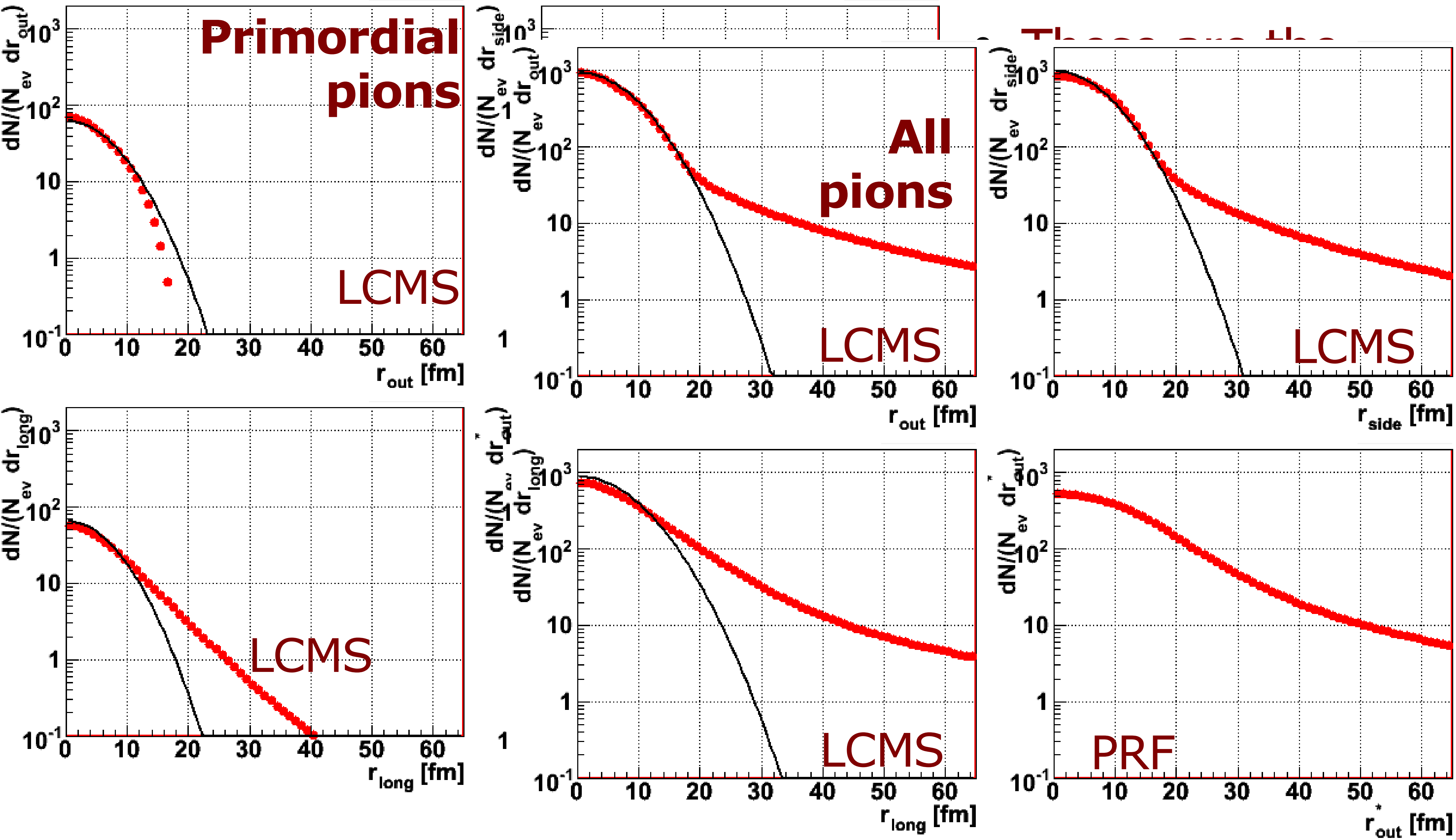
$$C(\vec{q}) = (1-\lambda) + \lambda K_{coul}(q_{inv})(1 + \exp(-R_{out}^2 q_{out}^2 - R_{side}^2 q_{side}^2 - R_{long}^2 q_{long}^2))$$

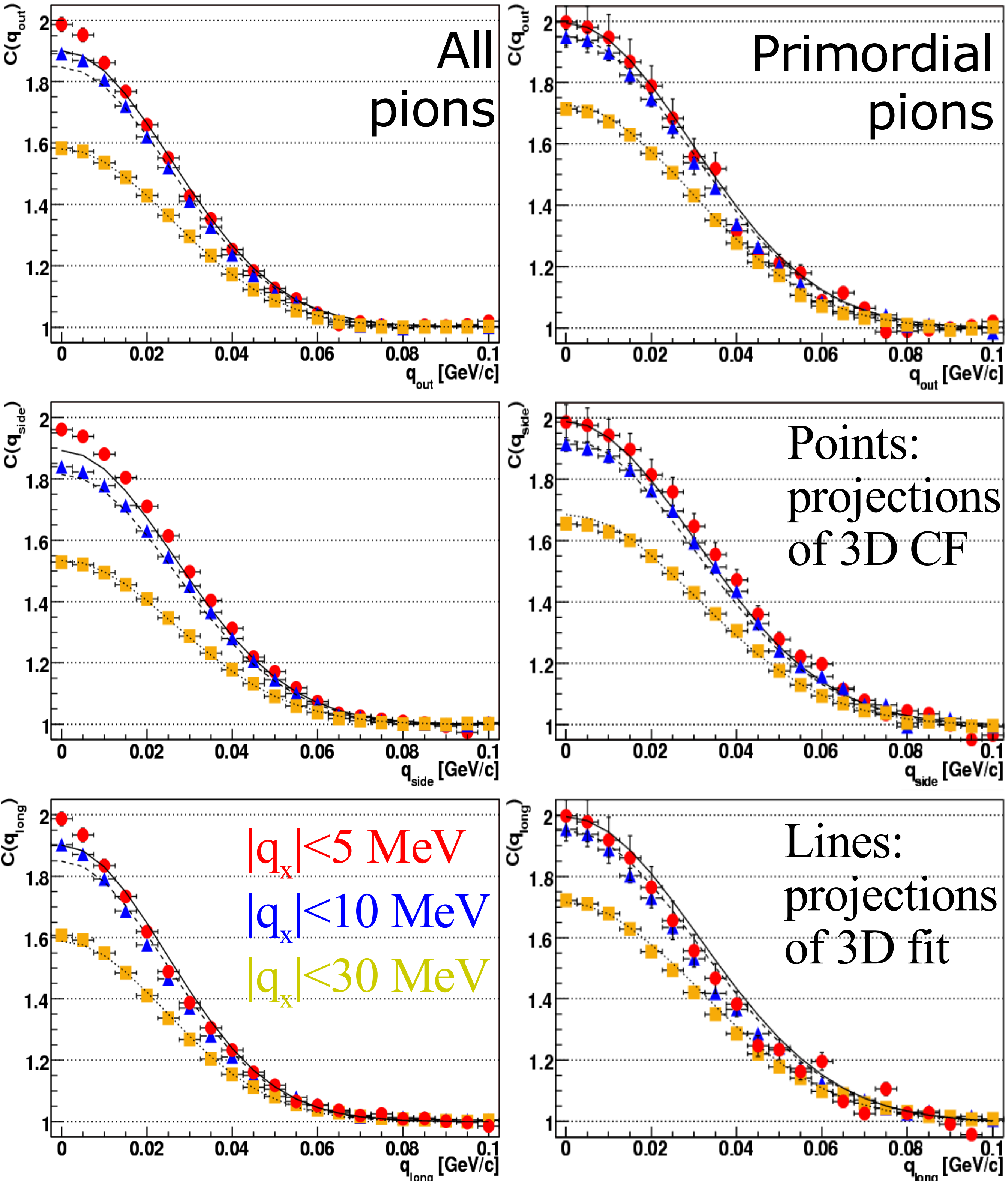
- K_{coul} is the Coulomb-only wave-function averaged over a 3D gaussian with the same, fixed size in all directions,.
- Following the experiment all analysis is done in LCMS

Single particle distributions



Two-particle separation distributions

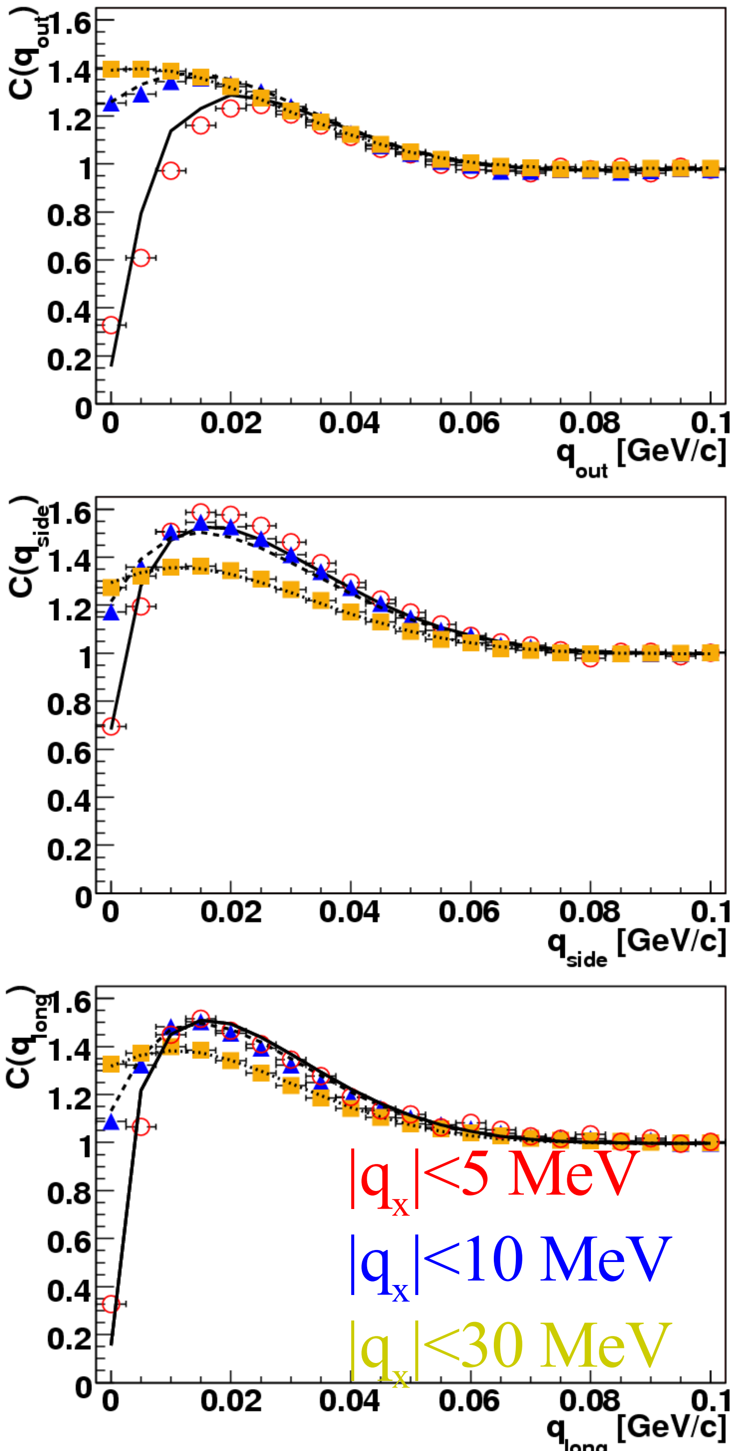




3D correlation functions

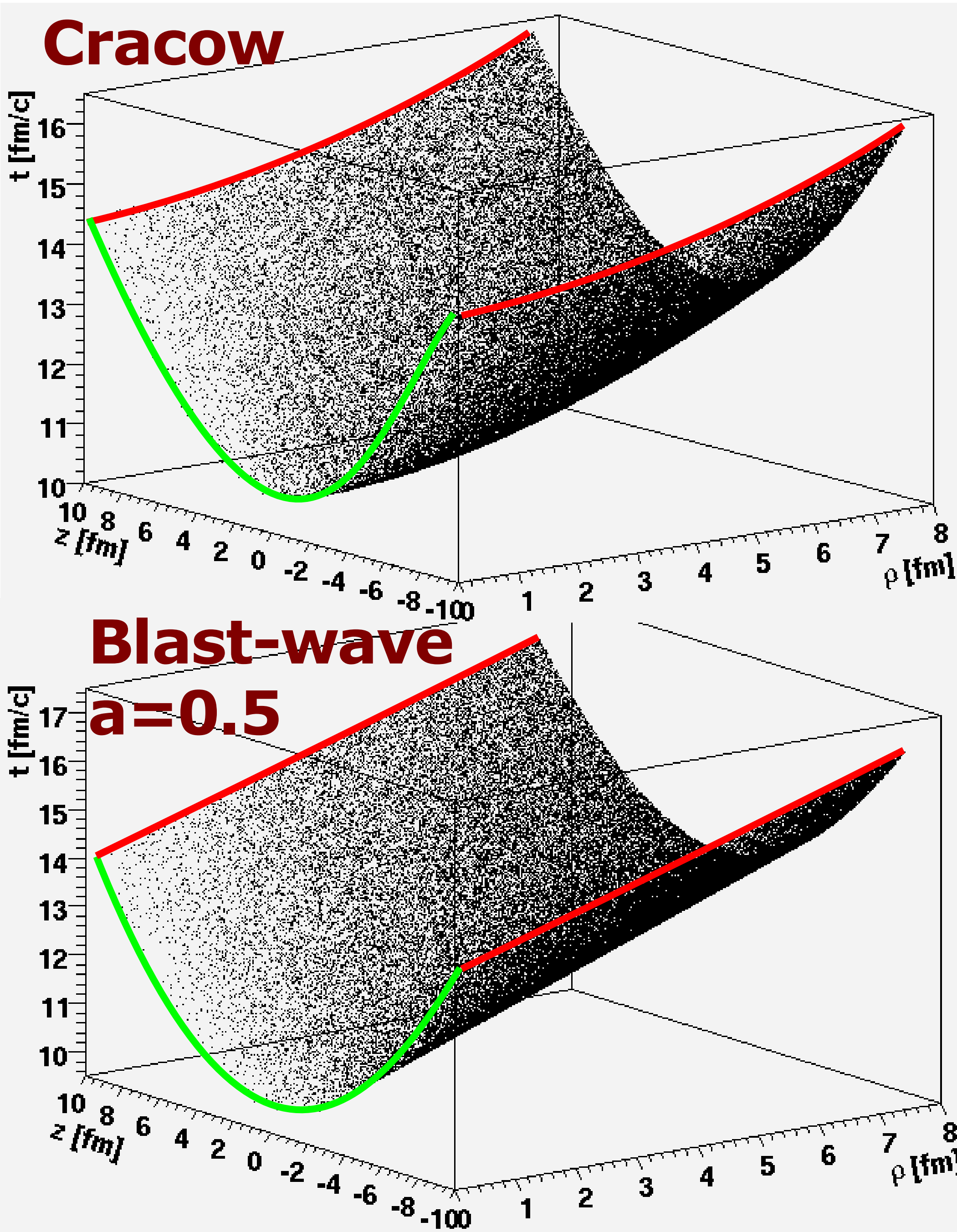
- We calculate 3D correlation functions and fit them with a traditional formula
- The fits perform relatively well, but the functions are significantly non gaussian (as expected!)

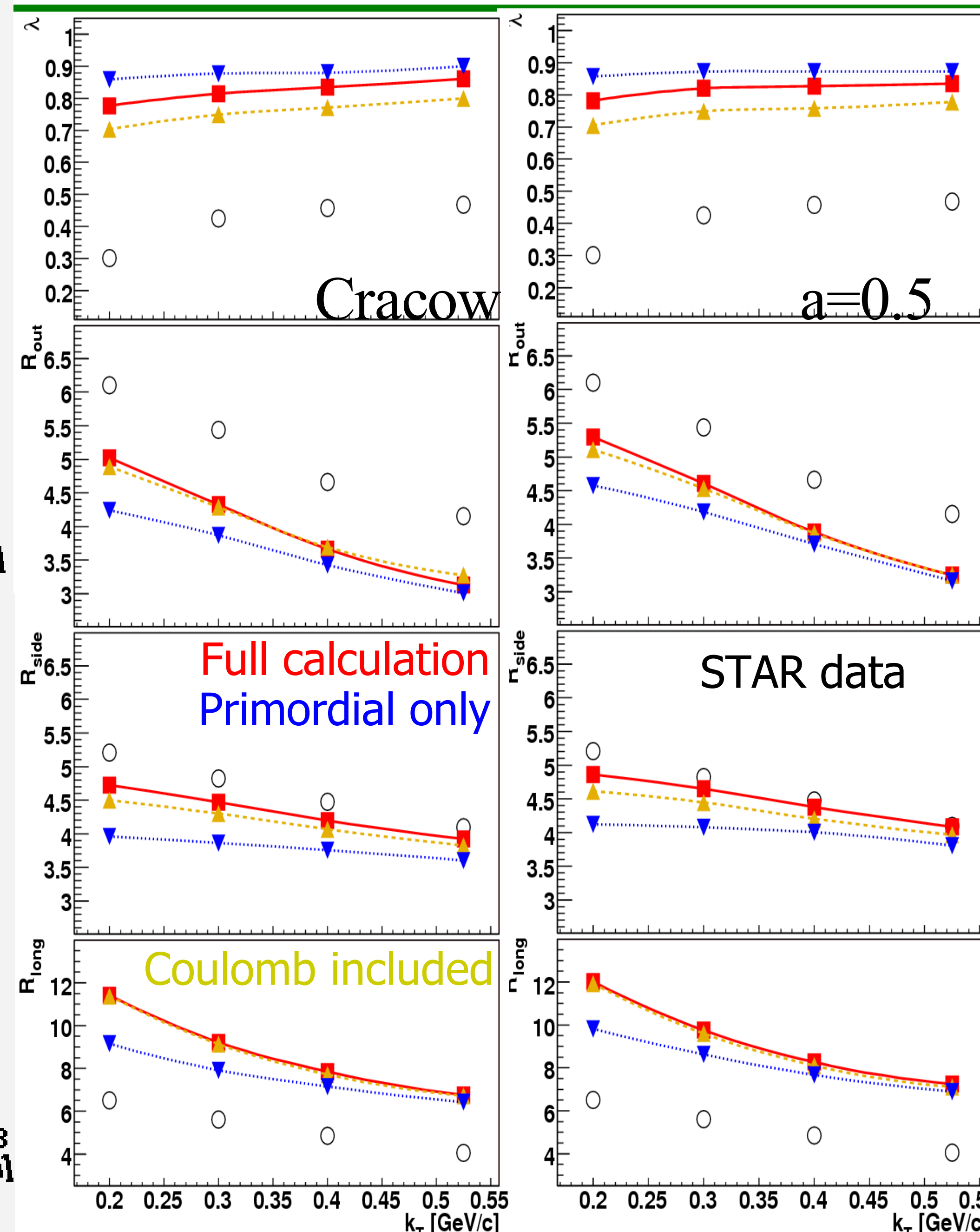
3D correlation functions with Coulomb



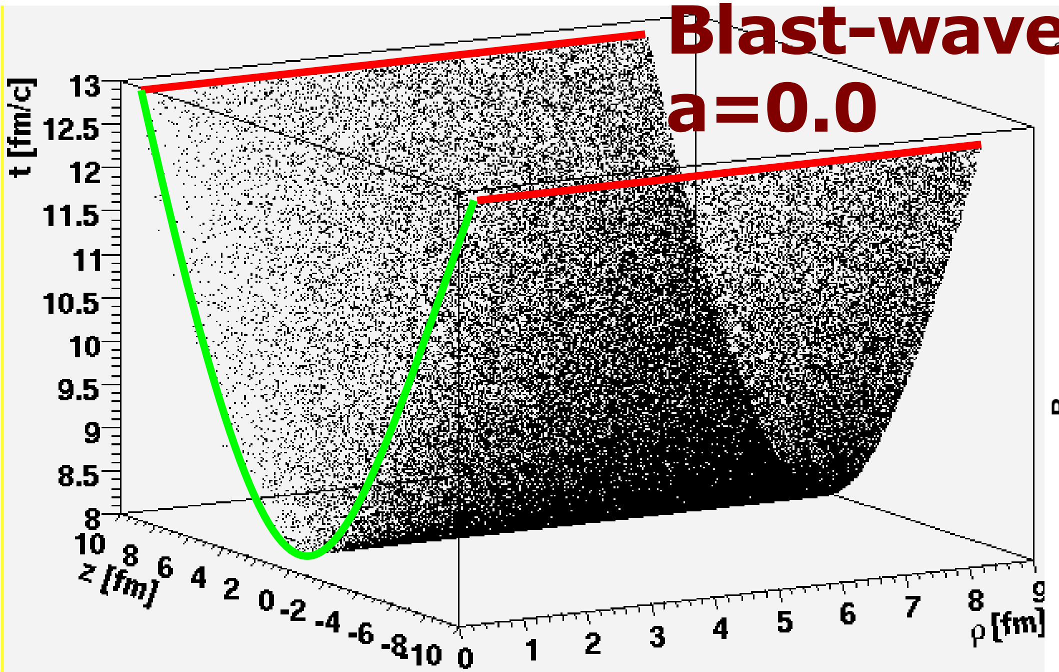
- The correlation function with full simulation of Coulomb effects can also be calculated
- It is fit with Bowler-Sinyukov formula. The fit is fully 3D. To plot it, we project it in the same manner as the input function.
- In this case we try to reproduce STAR data, therefore the K_{coul} for the spherical gaussian with radius 5fm in all directions was used.

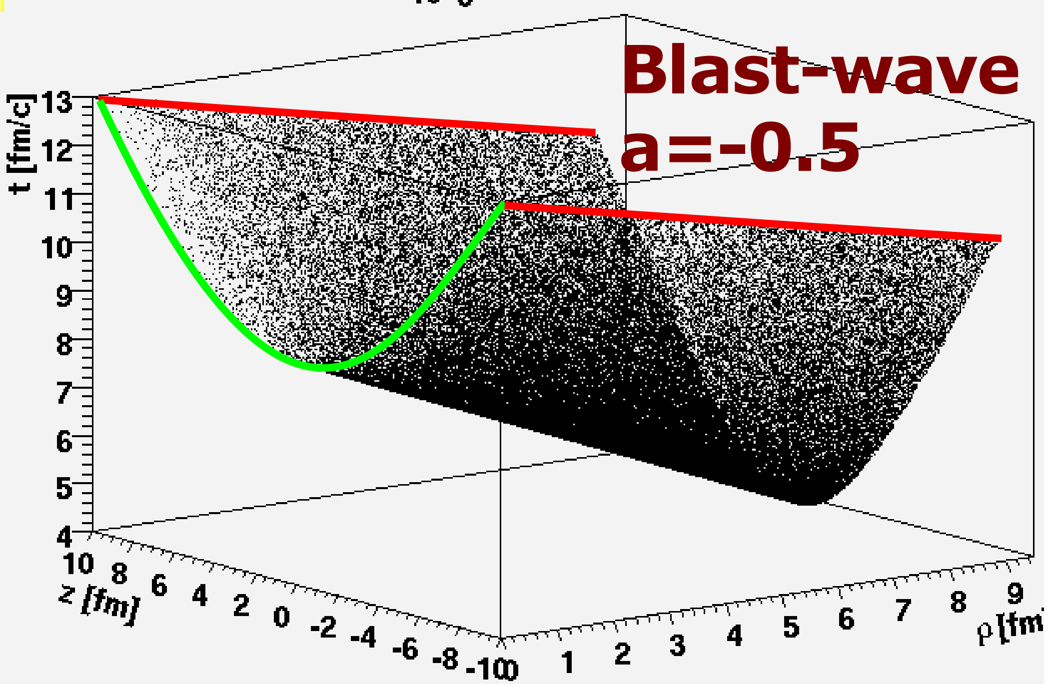
Comparing hypersurfaces

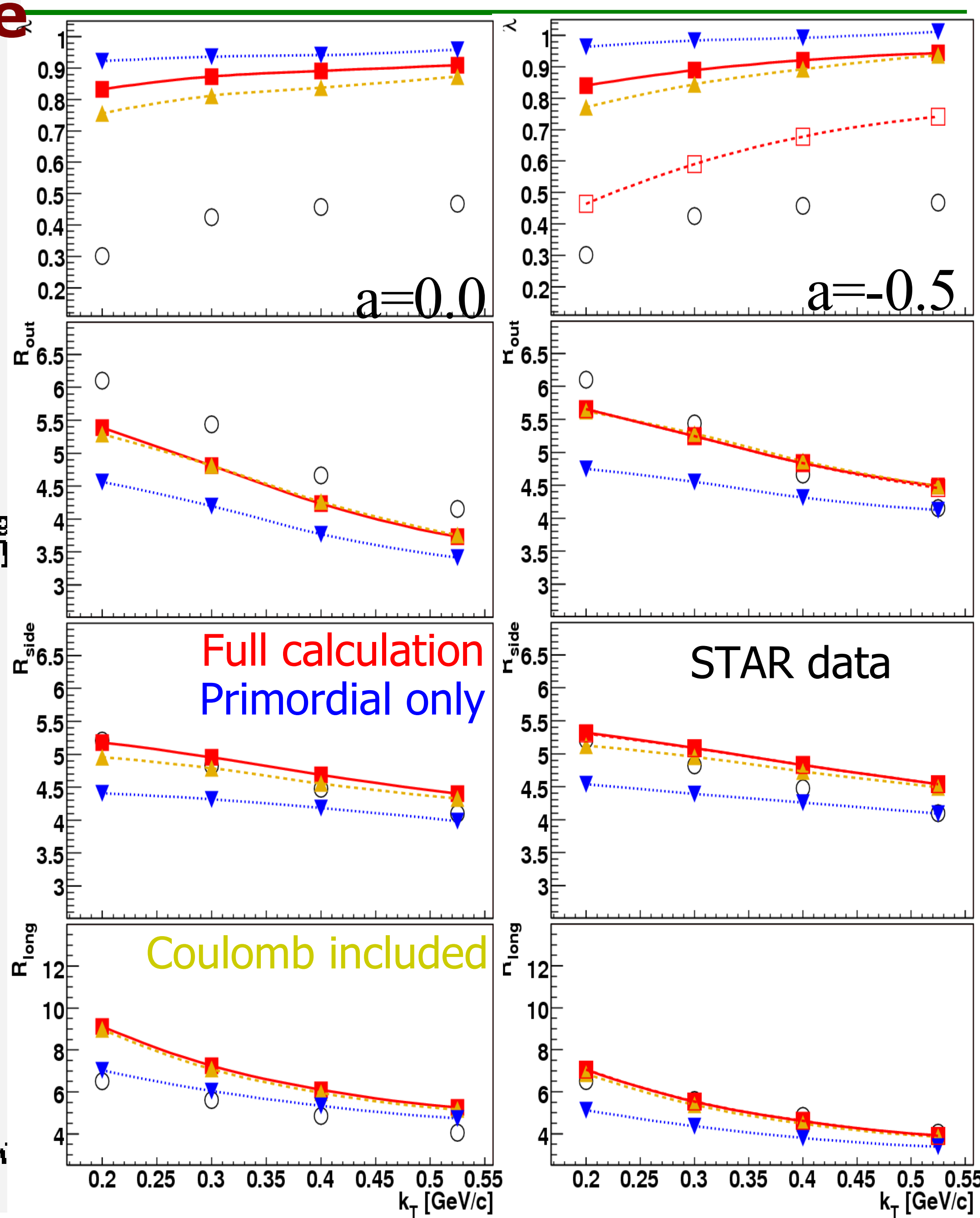


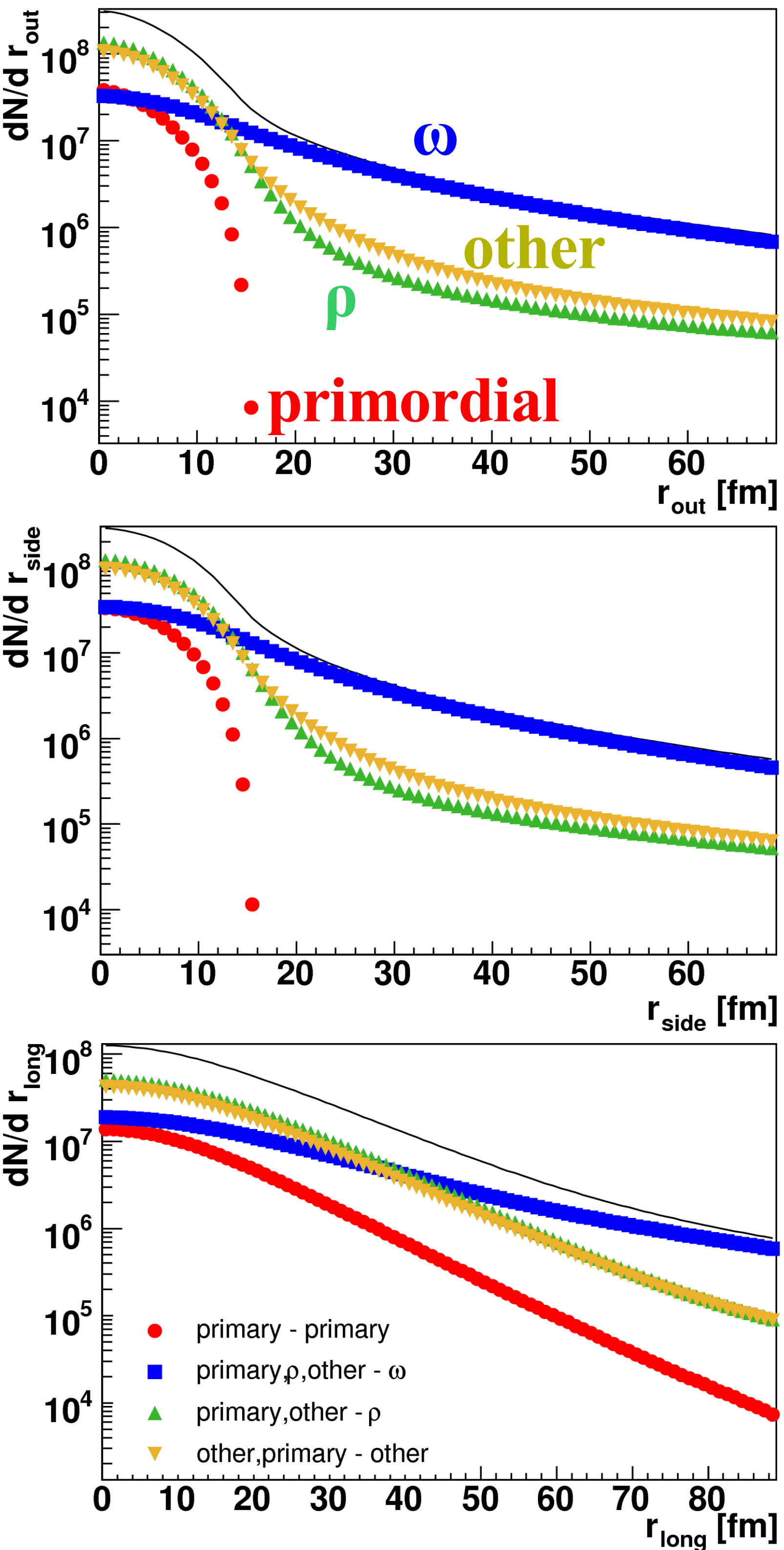


Modifying r-t slope





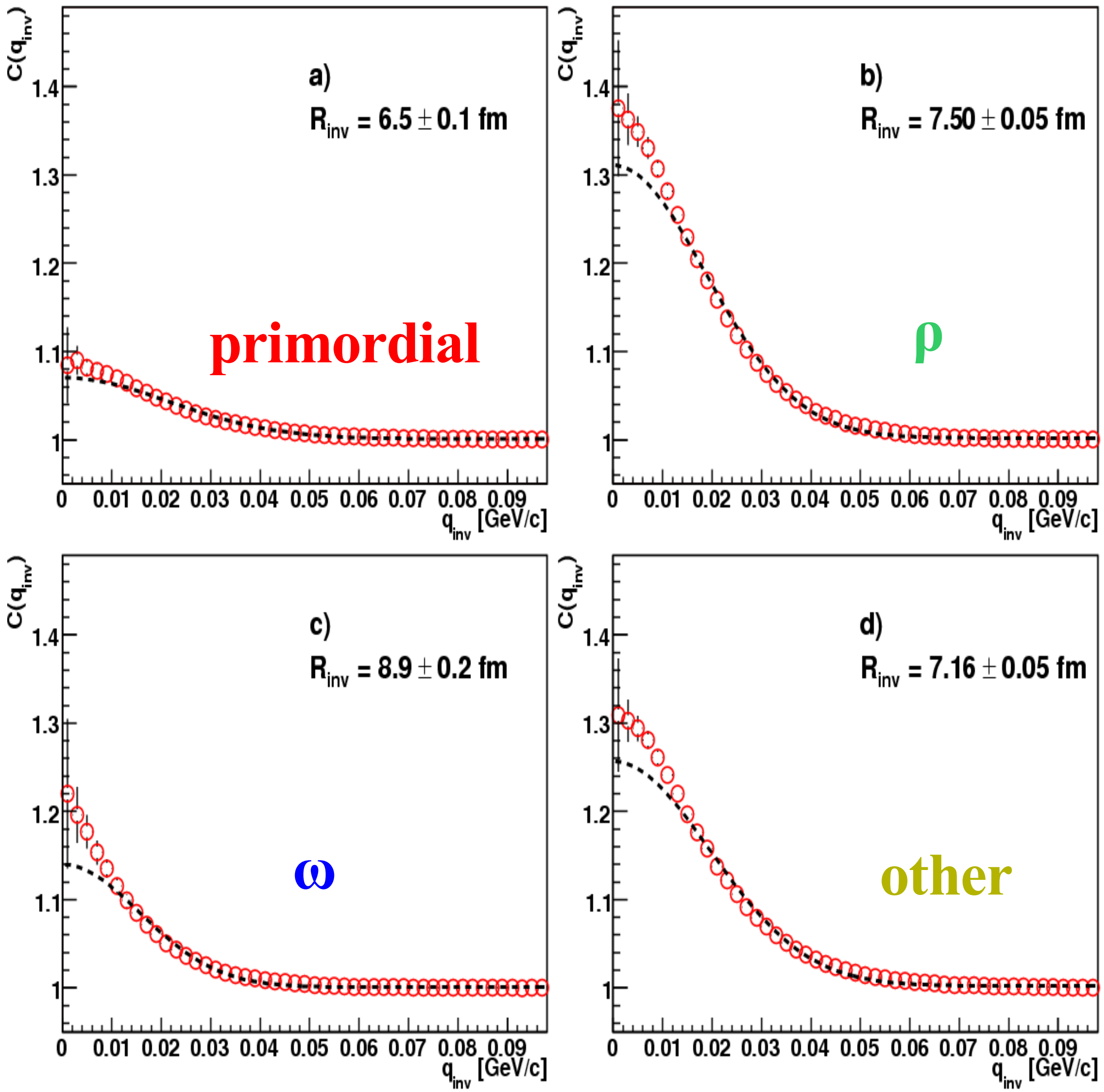




Resonance contribution

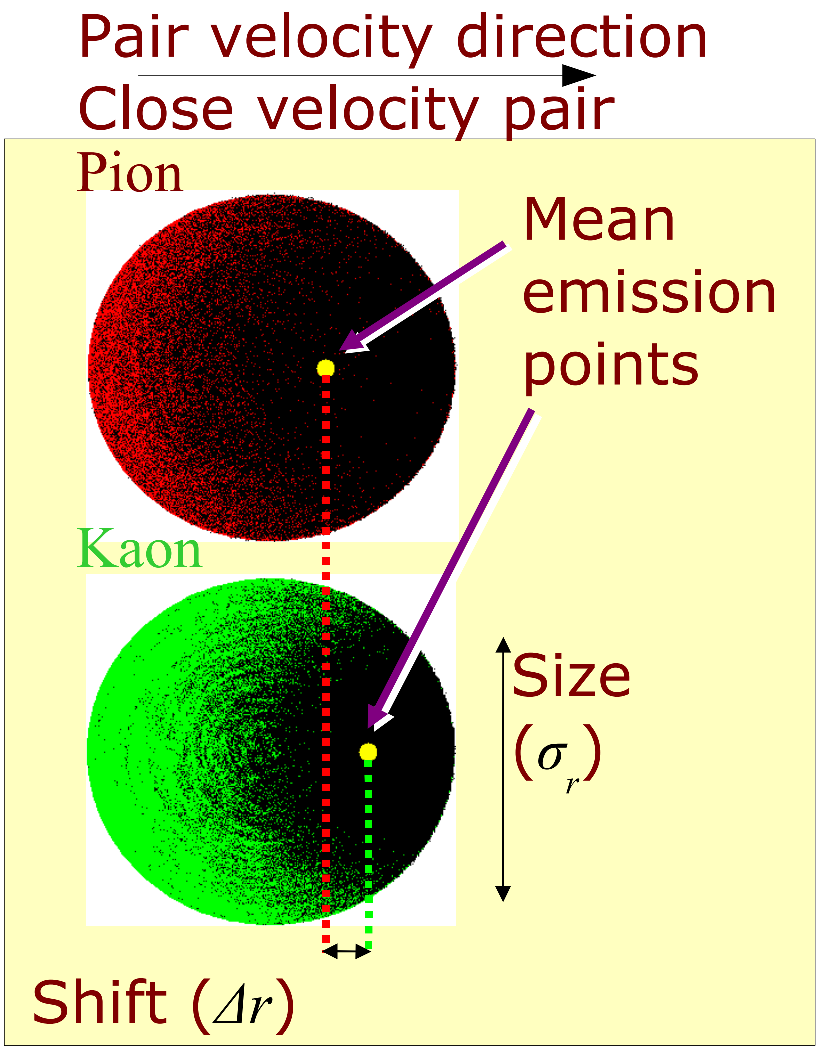
- Resonances have significant influence on the separation distributions – they produce long tails and enlarge the source
- The shape of the source is significantly non-gaussian
- The effect in the long direction is mixed with the influence of longitudinal expansion
- The source is completely non-gaussian in the long direction

Influence on the correlation function



- Functions non-gaussian, but width roughly reproduced
- Primordial pairs give only 10% of correlation effect
- Resonances increase the size by about 1fm
- Contribution from omega sharply peaked – mostly visible in the lambda parameter

Femtoscopy with Coulomb FSI



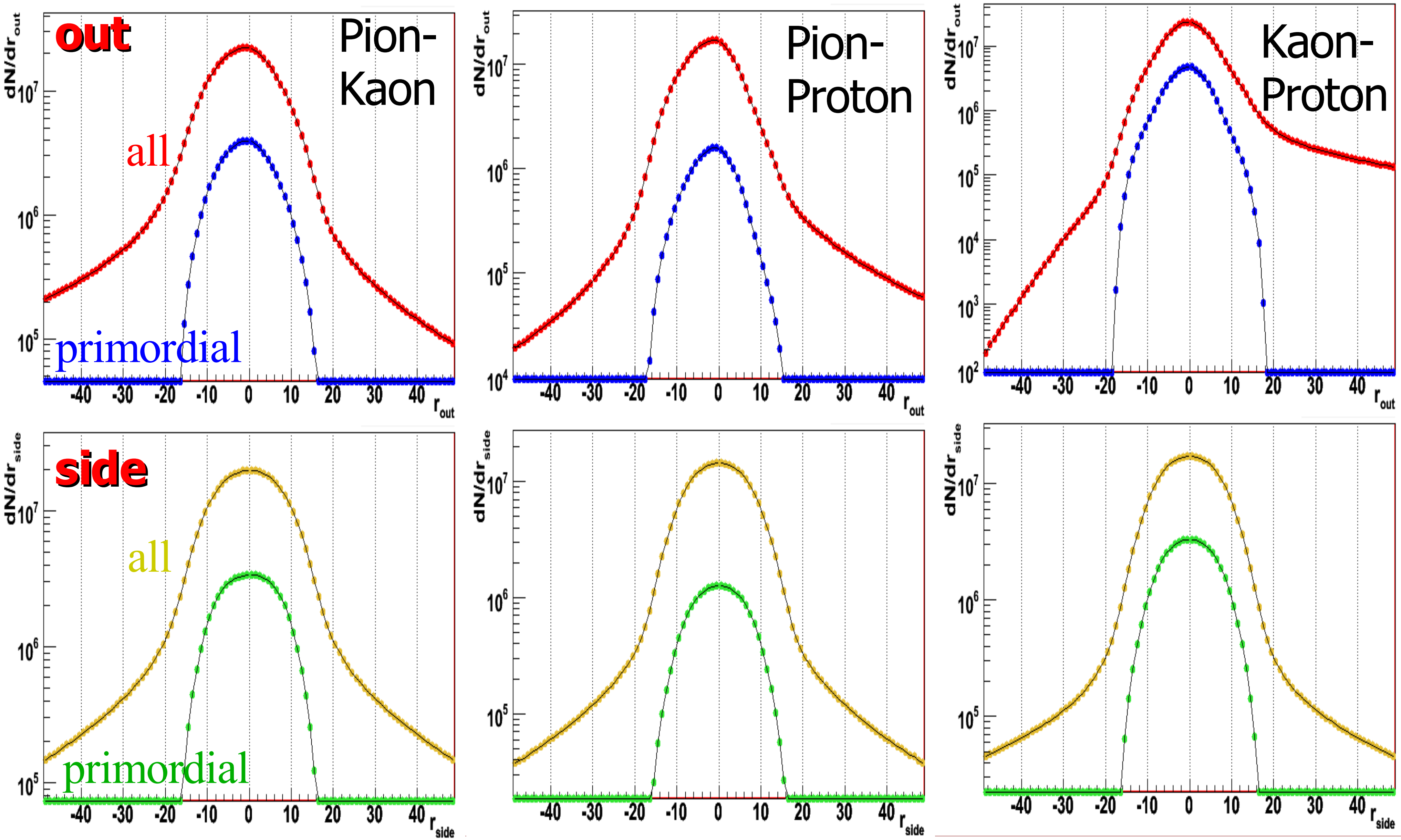
R.Lednicky et al.Phys.Lett. B373 (1996) 30.
S.Voloshin, R.Lednicky, S. Panitkin, N.Xu, Phys.Rev.Lett.**79**(1997)30

- Hydrodynamic calculations with radial flow predict two effects:
  - Size decreases with particle m_T (length of homogeneity)
  - Mean emission point is shifted from the center
- Space effects are mixed with time shift in PRF:

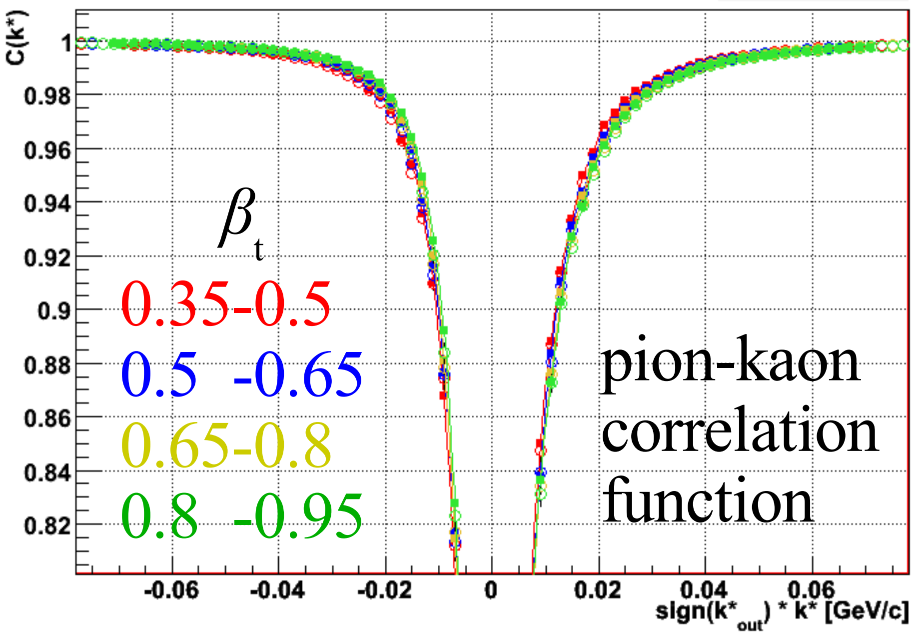
$$r^*_{out} = \gamma(\Delta r_{out} + \beta_T \Delta t)$$

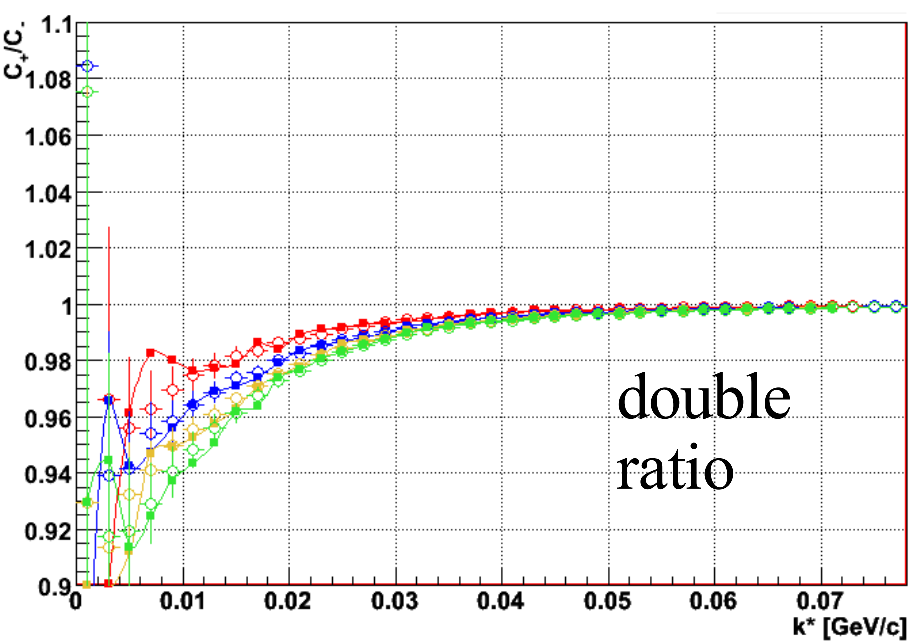
it is crucial to simulate resonance contribution in detail

Separation distributions for non-identical systems



Non-identical correlation functions



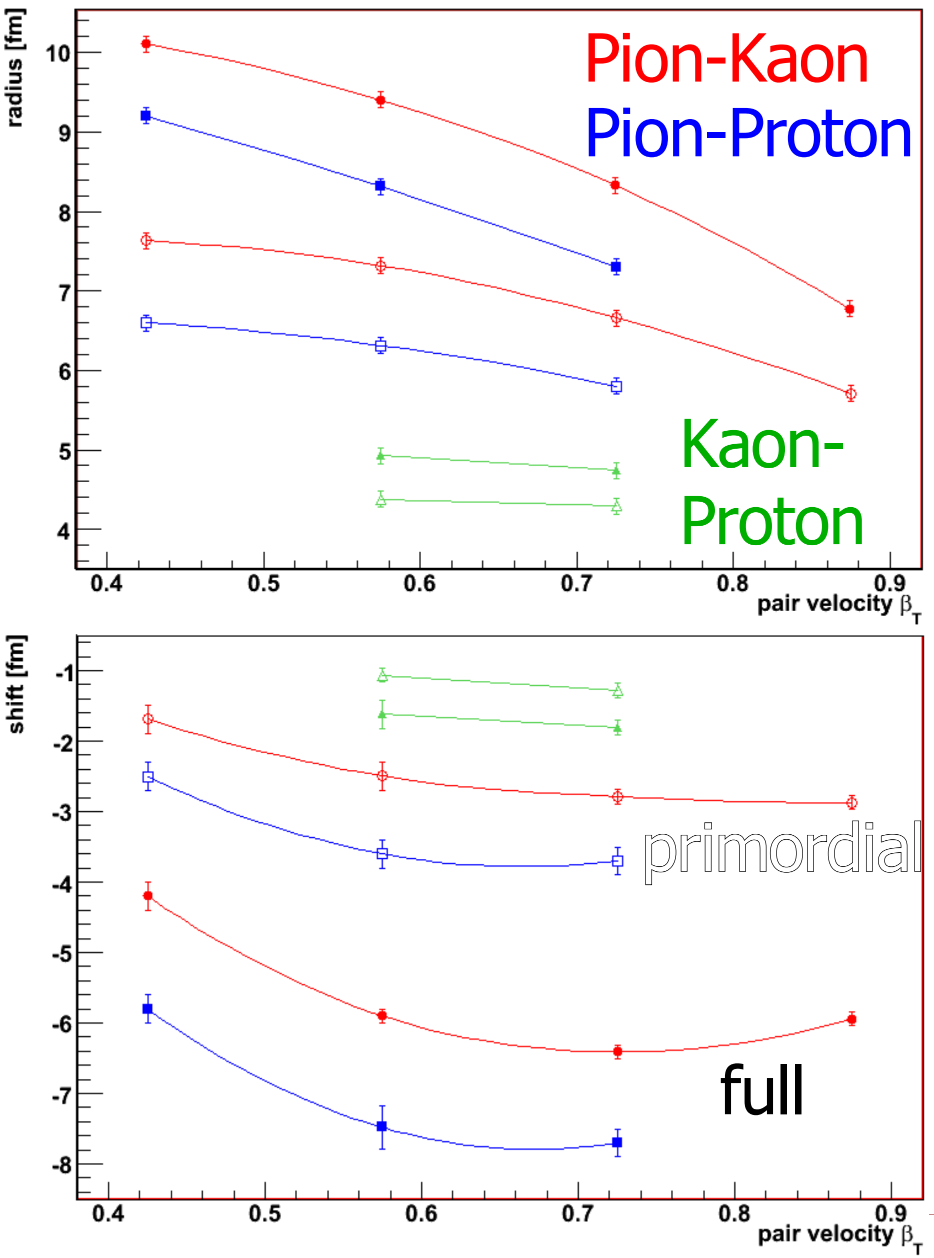


- We fit the model CF with a one simulated with the source function assumption:

$$S(\vec{r},\vec{K}) \sim \exp\left(-\frac{(r_{out}-\mu_{out})^2}{\sigma^2} - \frac{r_{side}^2}{\sigma^2} - \frac{r_{long}^2}{\sigma^2}\right)$$

- Only 2 parameters (σ, μ_{out}) can be independent if we fit 1D function
- The correlation strength gives the size, the asymmetry determines the shift of average emission point

Non-identical particles results



- We observe strong asymmetries in pion systems
- Resonance propagation has large influence on asymmetry
- We observe the m_T scaling of sizes similar to identical particle interferometry
- Relative abundances of resonance to primordial particles important in quantitative analysis

Summary

- “HBT radii” are obtained from the blast-wave type of models with all resonance included, by performing full 3D simulation of the correlation function, with the possibility to include Coulomb effects
- Comparison to STAR data shows the single freeze-out model to be consistent with data, and prefering the freeze-out hypersurface with negative r-t slope.
- Resonances are found to influence all aspects of the correlation function: enlarge the size, decrease lambda, produce non-gaussian tails in the pair distributions
- First results on non-identical particle correlations study were presented