Anomalous diffusion of pions at RHIC

T. Csörgő, M. Csanád and M. Nagy

MTA KFKI RMKI, Budapest, Hungary

Introduction:

- Normal diffusion
- Anomalous diffusion

Model simulation of anomalous diffusion of pions at RHIC

- PHENIX nucl-ex/0605032: evidence for a heavy tail of S(r)
- Selection of a model
- Rescattering effects
- Powerlaw tails
- Particle id dependence

Anomalous diffusion and a second order QCD phase transition:

future measurements: how to falsify these scenarios?

Discovering New Laws

"In general we look for a new law by the following process.
First we guess it.
Then we compare the consequences of the guess to see
what would be implied if this law that we guessed is right.
Then we compare the result of the computation to nature,
with experiment or experience, compare it directly with observation,
to see if it works.
If it disagrees with experiment it is wrong.

In that simple statement is the key to science.
It does not make any difference how beautiful your guess is.
It does not make any difference how smart you are,
who made the guess, or what his name is —
if it disagrees with experiment it is wrong."

/R.P. Feynman/

Normal diffusion and Gaussian sources

R. Metzler, J. Klafter / Physics Reports 339 (2000) 1–77

$$W_j(t+\Delta t) = \tfrac{1}{2} W_{j-1}(t) + \tfrac{1}{2} W_{j+1}(t)$$

$$\frac{\partial W}{\partial t} = K_1 \frac{\partial^2}{\partial x^2} W(x,t)$$

$$\frac{\partial W}{\partial t} = -K_1 k^2 W(k,t)\ ,$$

$$W(k,t) = \exp(-K_1 k^2 t)\ ,$$

$$W(x,t) = \frac{1}{\sqrt{4\pi K_1 t}} \exp\left(-\frac{x^2}{4K_1 t}\right)$$

$$W_0(x) \equiv \lim_{t\to 0+} W(x,t) = \delta(x).$$

$$W_{j\pm 1}(t) = W(x,t) \pm \Delta x \frac{\partial W}{\partial x} + \frac{(\Delta x)^2}{2} \frac{\partial^2 W}{\partial x^2} + \mathrm{O}([\Delta x]^3)$$

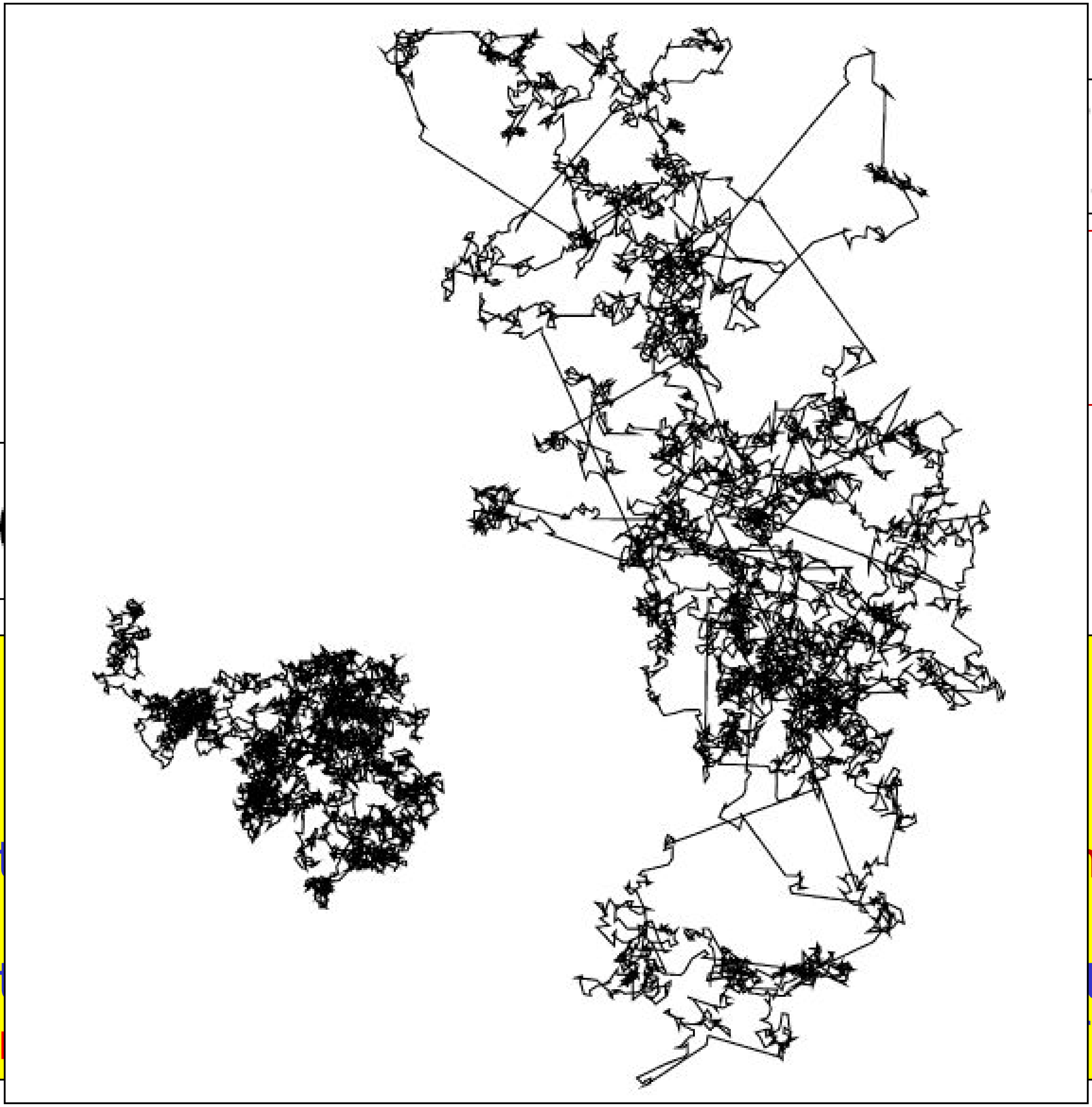
$$W_j(t+\Delta t) = W_j(t) + \Delta t \frac{\partial W_j}{\partial t} + \mathrm{O}([\Delta t]^2)$$

$$K_1 \equiv \lim_{\Delta x\to 0, \Delta t\to 0} \frac{(\Delta x)^2}{2\Delta t}$$

$$[K_1] = \mathrm{cm}^2\,\mathrm{s}^{-1}$$

$W_j(t)$ ~ S(x,t)
Jumps nearest neighbouring cells
j-1 -> j,   j+1 -> j
Master equation ~ Boltzmann equation
Diffusion equation
Scattering: time independent mean free path
-> corresponds to Gaussian random walk
normal (Gaussian) diffusion of pions
in homogeneous medium

Anomalous diffusion and Levy stable laws



$_{\infty}$ $\mathrm{d}x\,\psi(x,t)$

$W(x,v,t)$

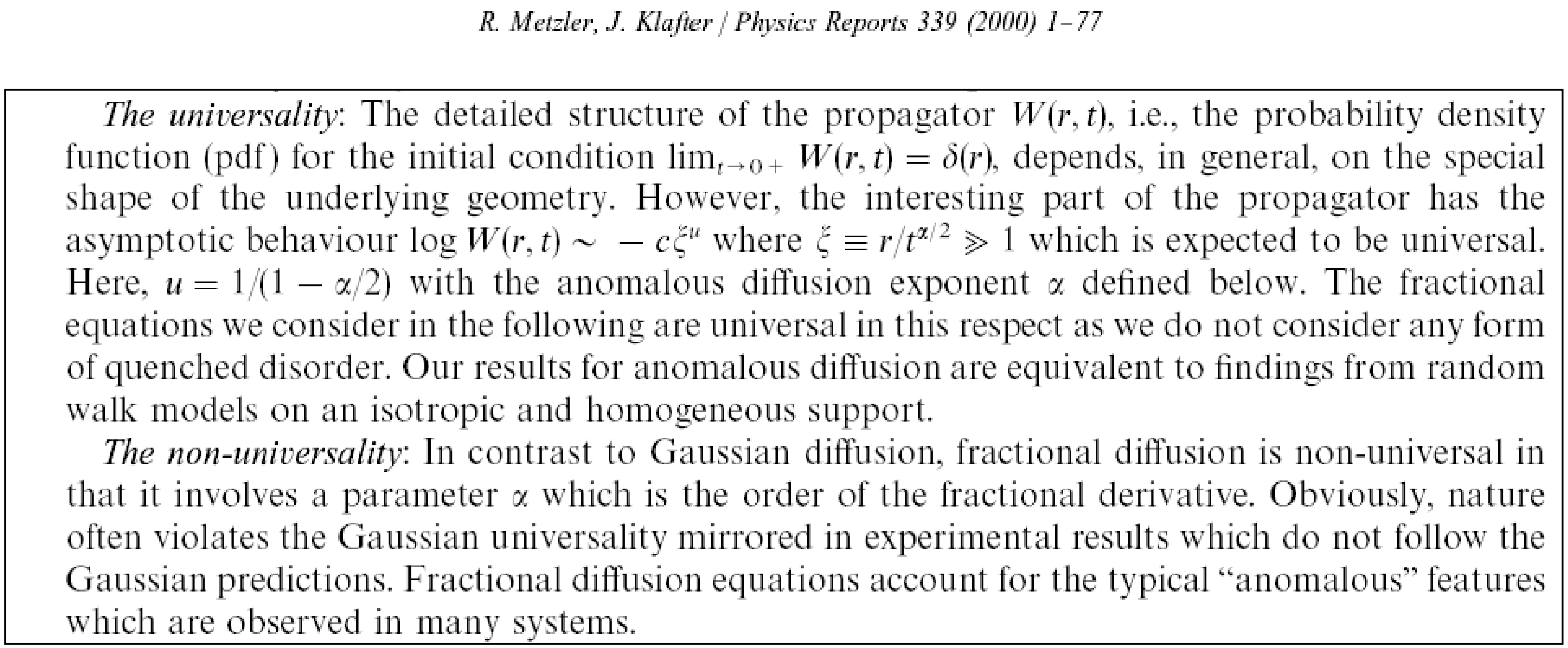
$W($

Rescat system

wit tions

anor ireball

Anomalous diffusion and Levy stable laws



R. Metzler, J. Klafter / Physics Reports 339 (2000) 1–77

The universality: The detailed structure of the propagator $W(r,t)$, i.e., the probability density function (pdf) for the initial condition $\lim_{t\to 0+} W(r,t) = \delta(r)$, depends, in general, on the special shape of the underlying geometry. However, the interesting part of the propagator has the asymptotic behaviour $\log W(r,t) \sim -c\xi^u$ where $\xi \equiv r/t^{\alpha/2} \gg 1$ which is expected to be universal. Here, $u = 1/(1-\alpha/2)$ with the anomalous diffusion exponent α defined below. The fractional equations we consider in the following are universal in this respect as we do not consider any form of quenched disorder. Our results for anomalous diffusion are equivalent to findings from random walk models on an isotropic and homogeneous support.

The non-universality: In contrast to Gaussian diffusion, fractional diffusion is non-universal in that it involves a parameter α which is the order of the fractional derivative. Obviously, nature often violates the Gaussian universality mirrored in experimental results which do not follow the Gaussian predictions. Fractional diffusion equations account for the typical "anomalous" features which are observed in many systems.

Relevant mathematical tools:
Generalized Boltzmann equation
Rescattering in a time dependent mean free path system
-> corresponds to random Levy walk
but let us look for a realistic model first

Selection of the MC model

- Conventional hadronic cascade model

- Describes single particle spectra
- Describes elliptic flow data
- Describes HBT data [w/o any puzzle]
    hence yields a good description of the hadronic final state i.e. an acceptable model of S(r) !

- Well documented and easy to use
- Works at CERN SPS as well as at RHIC energies

- Contains the most important
    short and long lived resonances
    e.g. ω, η, η' (halo of long-lived resonances)
    AND includes their rescattering in the code

HRC and exact Buda-Lund hydro solutions

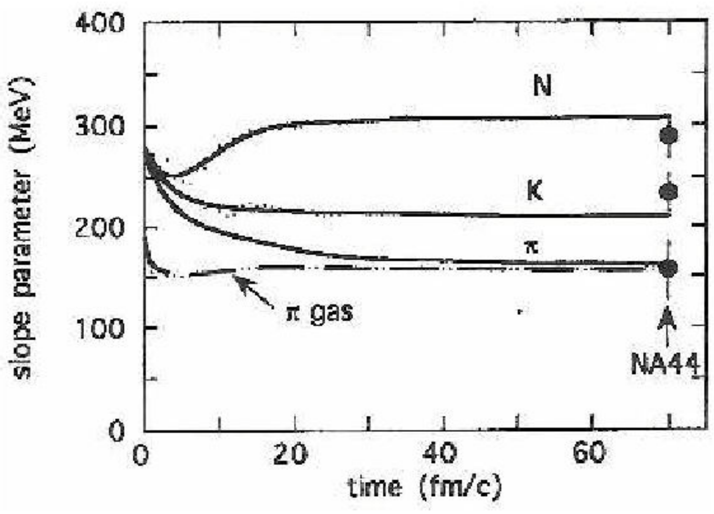
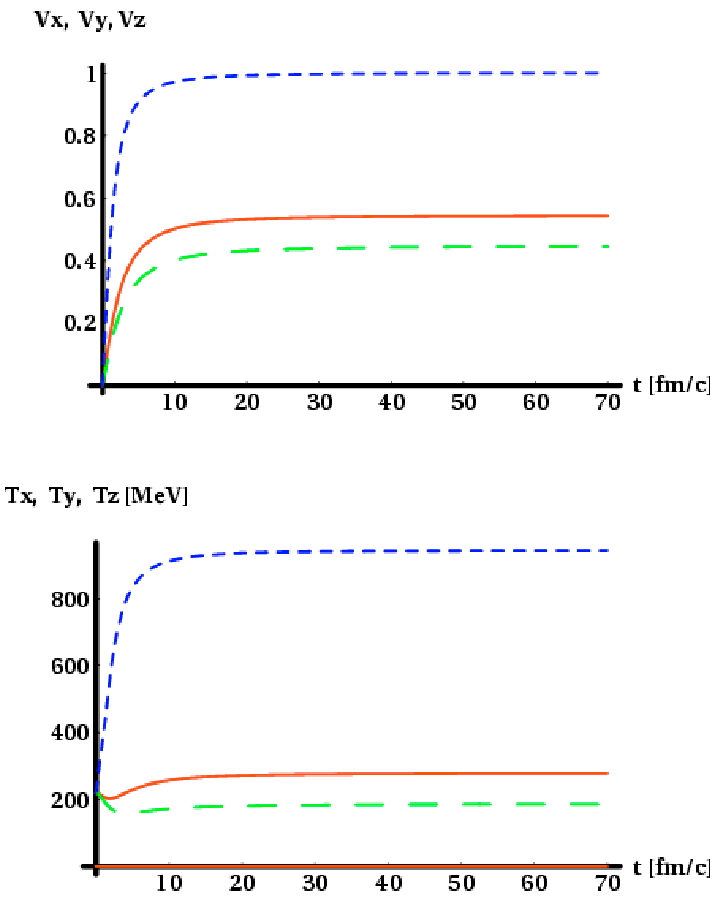
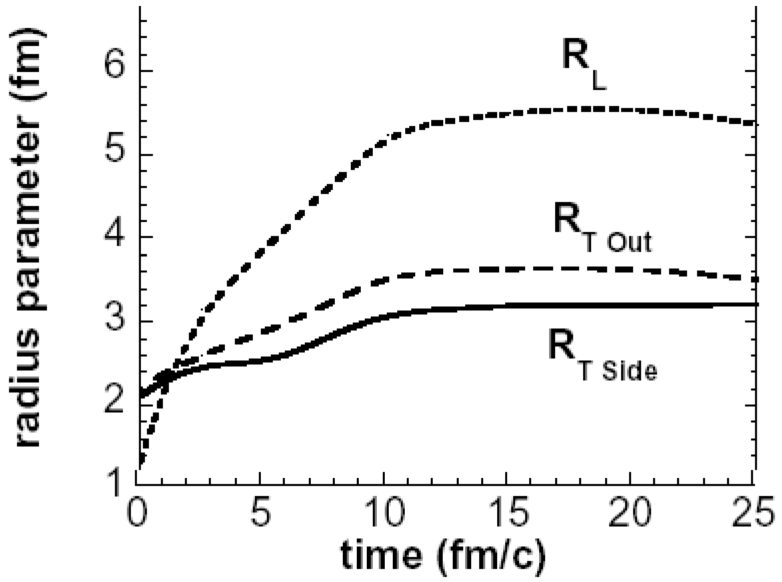


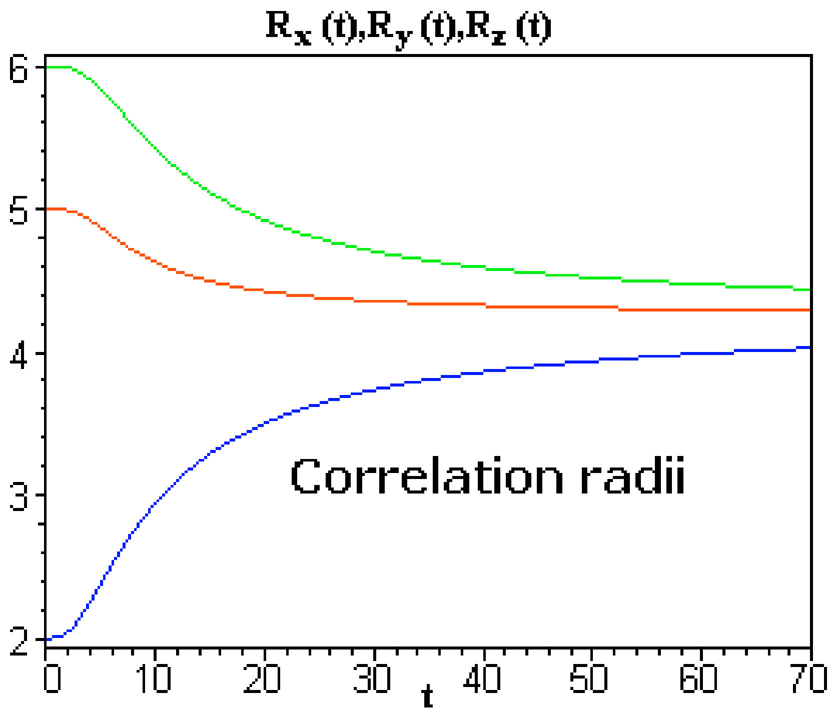
Fig. 16. Time evolution of the slope parameters in HRM for SPS Pb+Pb collisions for a full calculation (solid lines) and for a "pion gas" calculation (dashed line). Experimental slope parameters from NA44 (points) are plotted at $t = 70$ fm/c for comparison (From Ref. [21]).

Hadronic Rescattering Model (HRM): first use of the word „shooting" when predicting spectra.
HRM and Buda-Lund hydrodynamical calculations: self-quenching effect.



HRC and exact Buda-Lund hydro solutions: HBT





HRM and Buda-Lund hydrodynamical calculations: self-quenching effect.
HBT radii stop to evolve in time, although the system keeps on expanding
(rescatterings or hydrodynamical evolution).

HRC and exact Buda-Lund hydro solutions: v2

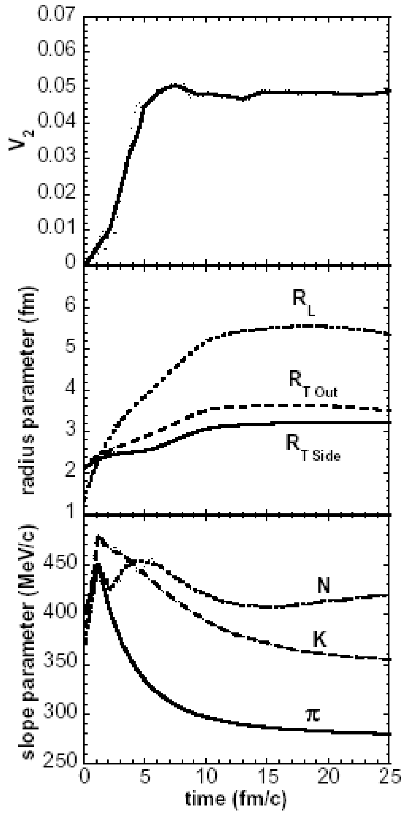
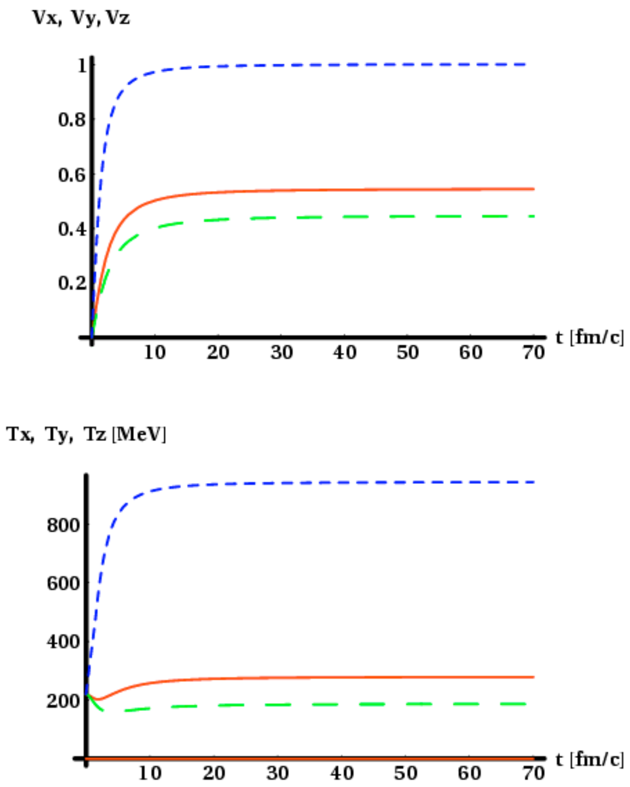
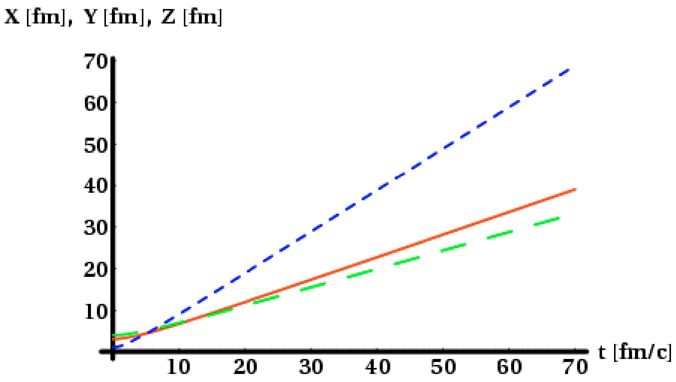


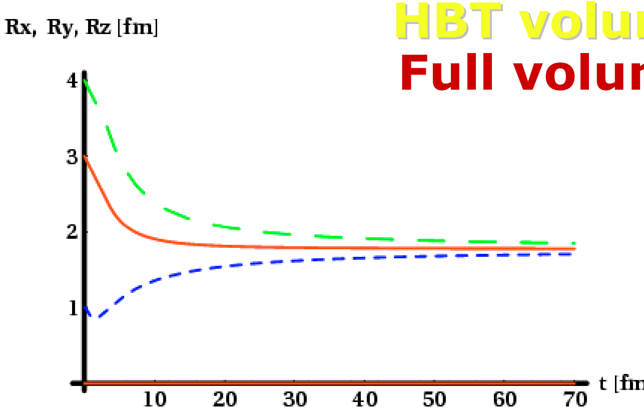
Fig. 18. Time evolution of V_2, HBT, and m_T slope parameters from HRM for RHIC Au+Au.

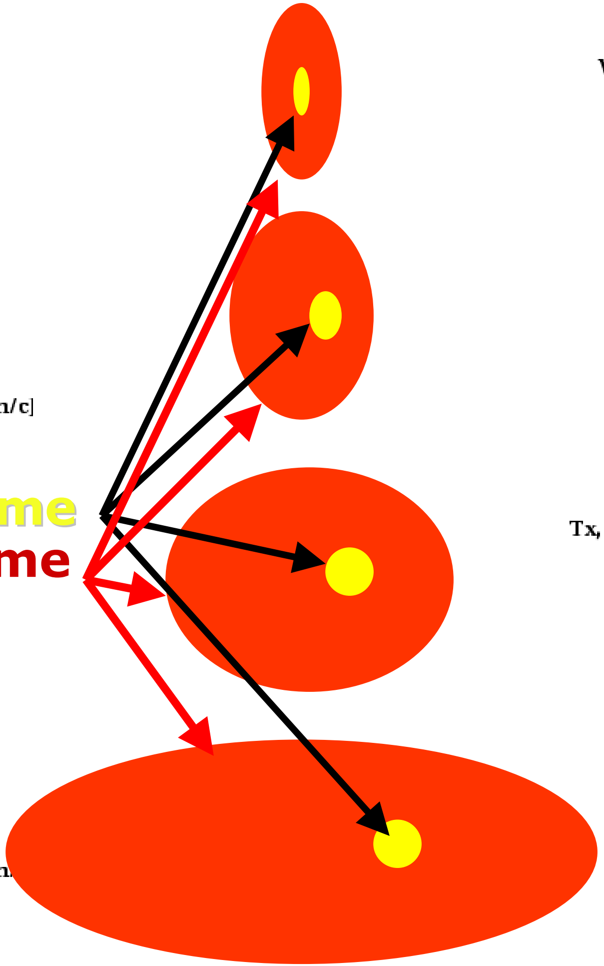


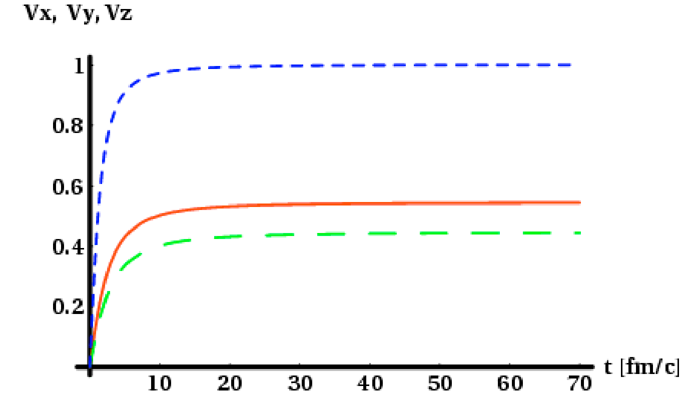
HRM and Buda-Lund hydrodynamical calculations: self-quenching effect. Elliptic flow freezes out at the same time when the spectra (slopes) stop to evolve in time, although the system keeps on expanding (rescatterings or hydrodynamical evolution).

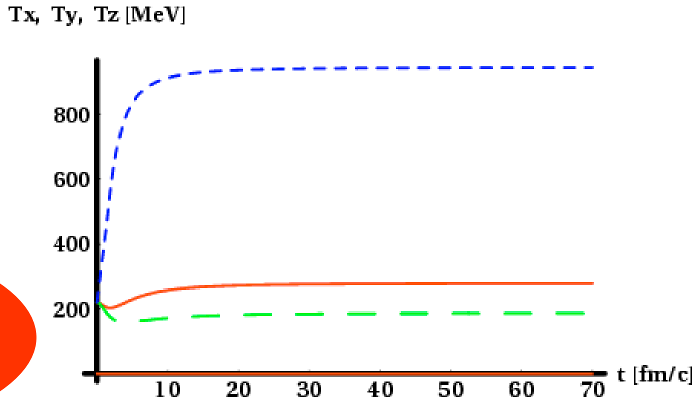
Solution of the “HBT puzzle”











Geometrical sizes keep on increasing. Expansion velocities tend to constants.
HBT radii R_x, R_y, R_z approach a direction independent constant.
Slope parameters tend to direction dependent constants.
General property, independent of initial conditions - a beautiful exact result.

Understanding hydro & HRC

New exact solutions of 3d nonrelativistic hydrodynamics+

HRC calculations: time evolution similar to a shot!

Hydro: **Shot of an arrow:**

Desription of data ⟷ Hitting the target

Initial c… velocity

Equatio… ntial

Freeze-… t

Data cor… lls

… tial (?)

Differen… n

exactly … aneously (!)

EoS and … n be

… e potential

Universal scaling of v_2 ⟷ In a perfect shot, the shape of trajectory is a parabola

Viscosity effects ⟷ Drag force of air

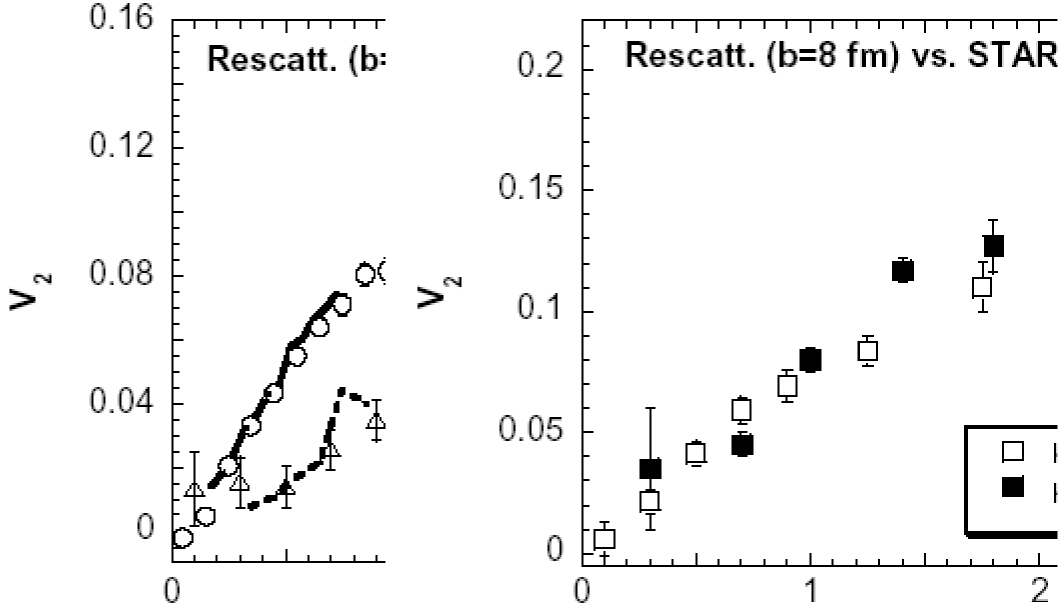
More than hydro: tails of particle production

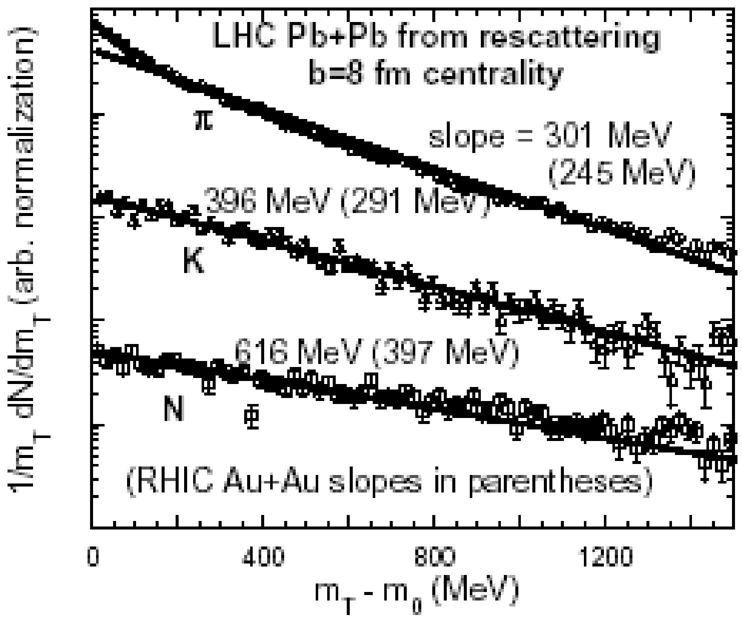
What about the rescattering after
hydrodynamics freezes out?

How to distinguish
hydro + ideal freeze-out (Therminator)

from cascades and
hydro+cascade scenario?

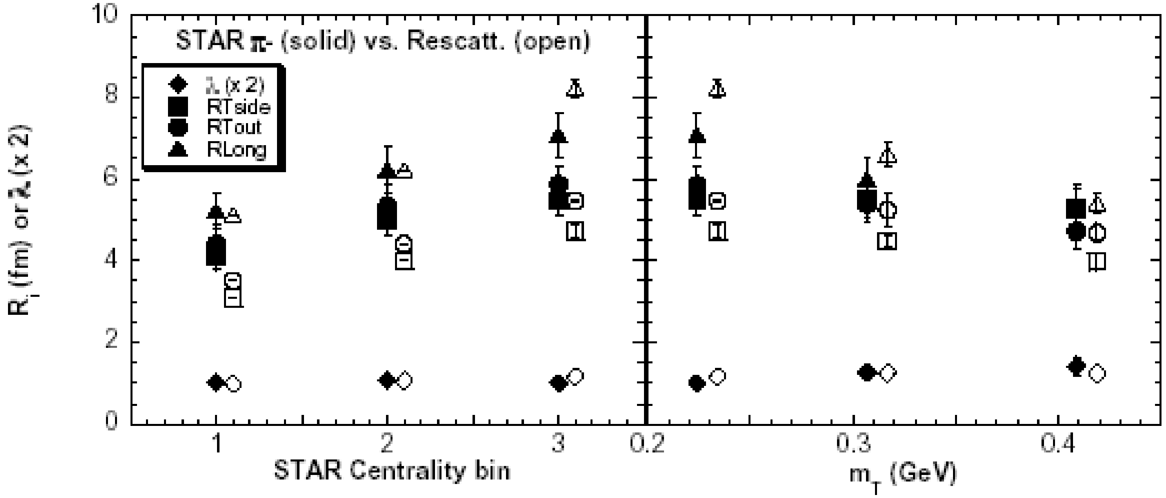
Demonstration: Model vs Au+Au 200GeV





Hadronic Rescattering Model (HRM):
Tom Humanic,
reviewed in
nucl-th/0510049
Int.J.Mod.Phys.E15:197-236,2006

OK at the freeze-out



Important properties of Tom's HRC

- Conventional hadronic cascade model

- Has an adaptive bin size in the time direction:
  hence no natural time scale in the code

- Contains the cascading of the most abundant hadrons:
  direct π

ρ	$\Gamma(\rho)$	= 152 MeV
Δ	$\Gamma(\Delta)$	= 120 MeV
K^*	$\Gamma(K^*)$	= 50 MeV
ω	$\Gamma(\omega)$	= 8.4 MeV
η	$\Gamma(\eta)$	= 0.0012 MeV
η'	$\Gamma(\eta')$	= 0.200 MeV
ϕ	$\Gamma(\eta')$	= 4.4 MeV
Λ	$\Gamma(\eta')$	= $2.5\text{x}10^{-12}$ MeV

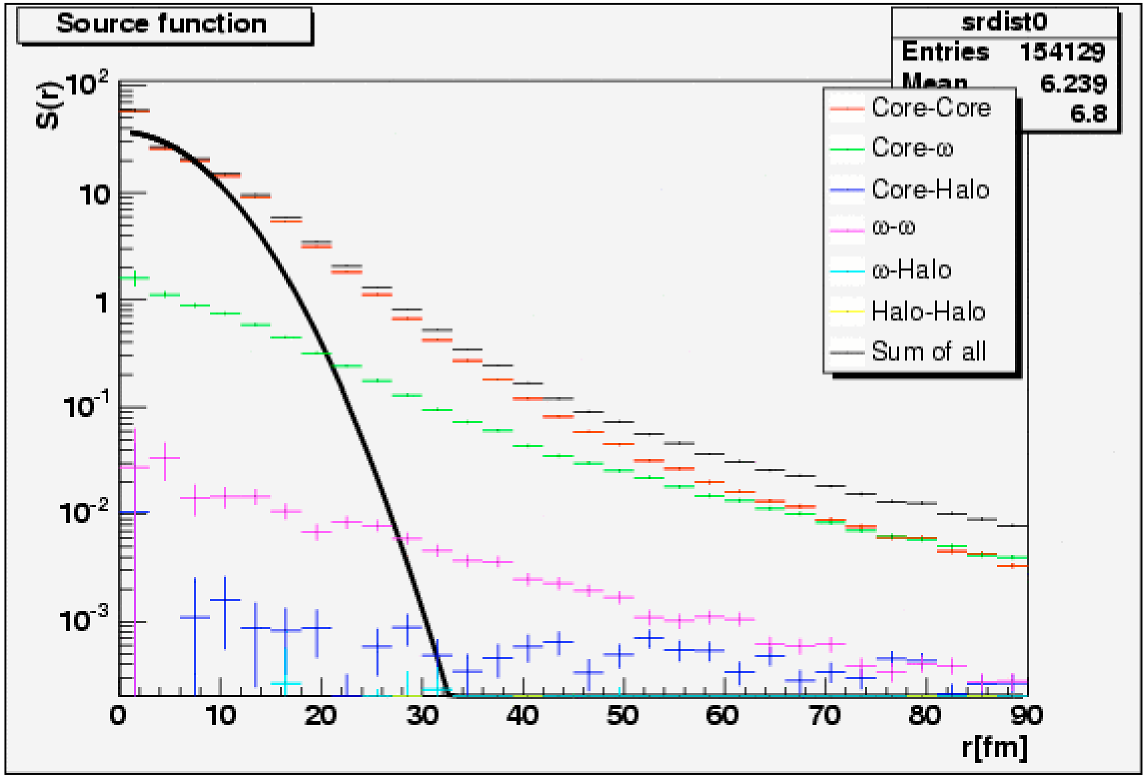
core: $\hbar/\Gamma < 4$ fm

ω: $\hbar/\Gamma = 23.5$ fm

halo: $\hbar/\Gamma > 40$ fm

- but neglect of electric charge

Tom’s HRC:
cuts for Fig.1 of nucl-ex/0605032

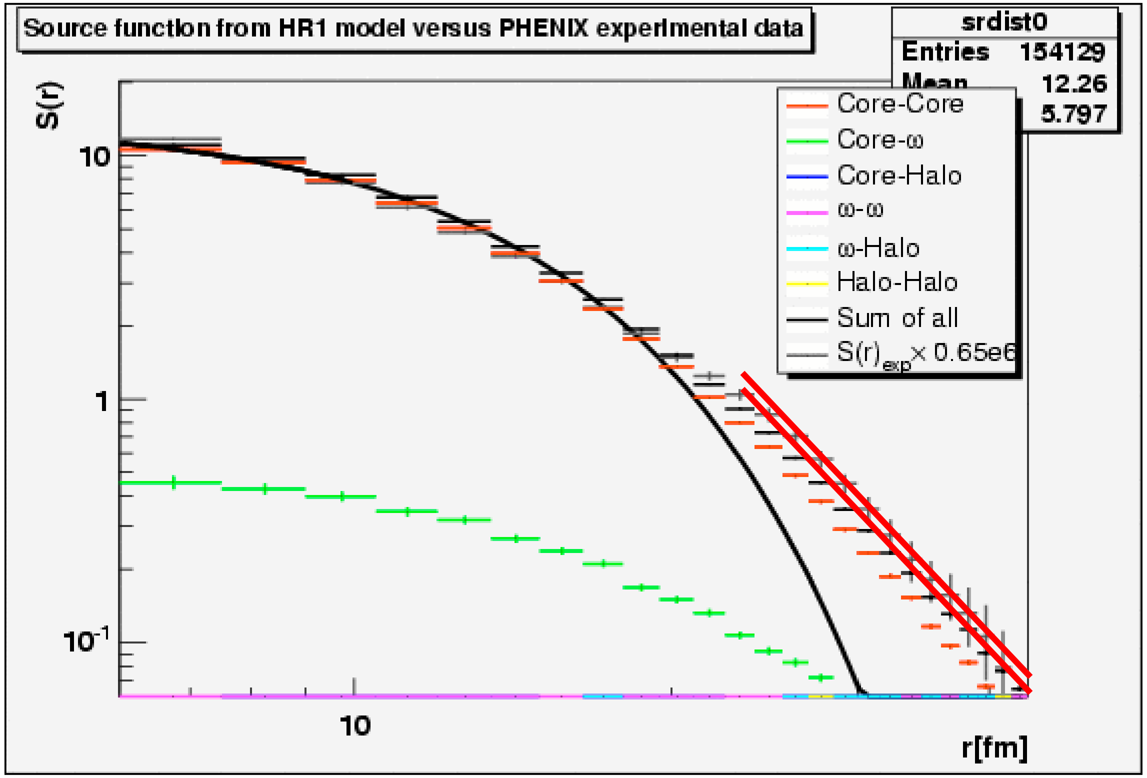


Gaussian to first few points
fails
at heavy tail
core-core pairs
reproduce heavy tail
omega-core pairs: small
dominate for r > 80 fm
only
all other pairs have
negligible contribution

Au+Au 200 GeV
0-20 % centrality
-0.5 < y < 0.5
0.20 GeV < kT < 0.36 GeV
pions (no charge selection)

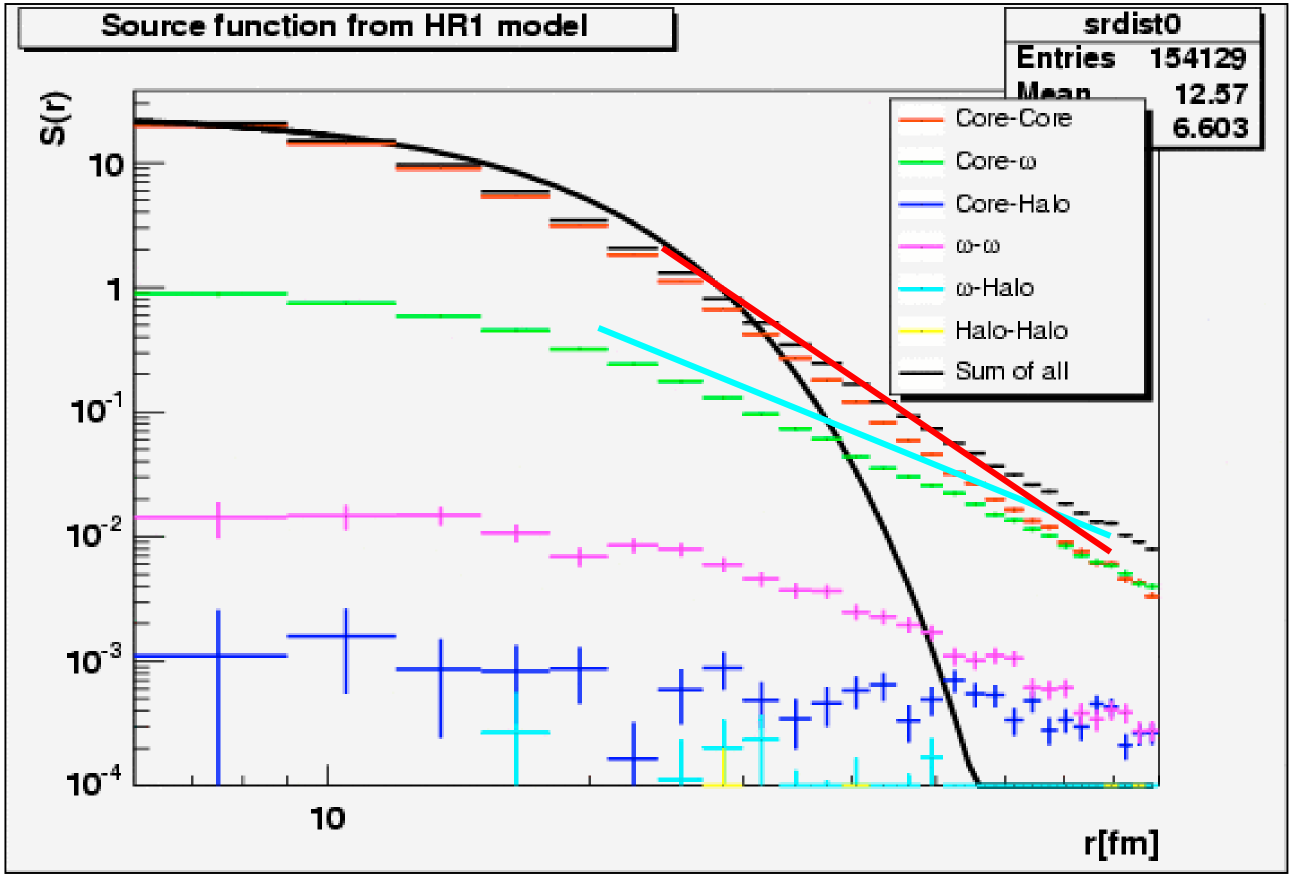
	core	ω	halo
core	x	x	x
ω		x	x
halo			x

nucl-ex/0605032 Fig.1 b and HRM, log-log scale



A power-law tail
due to rescattering (in HRM)
-> corresponds to Levy distributions
Reason: adaptive scale in HRM, corresponding to
anomalous diffusion (in contrast to Gaussian diffusion)

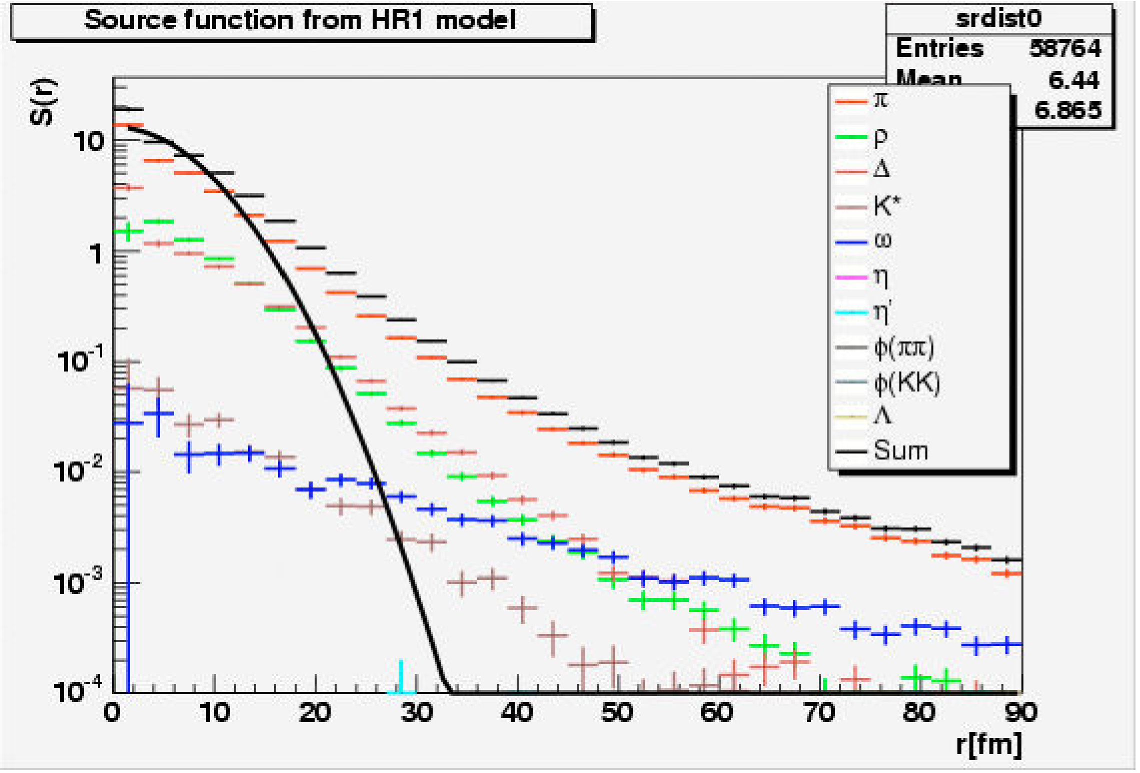
Same plot, on a larger, log-log scale



Prediction: at a larger scale power-law tail
has two components: core-core and omega-core
-> corresponds to sum of Levy distributions
but rescattering is dominant (core-core) for $r < 80$ fm in HRC

More detailed decomposition

	π	ρ	Δ	K*	ω	η	η'	ϕ
π	x							
ρ		x						
Δ			x					
K*				x				
ω					x			
η						x		
η'							x	
ϕ								x



Now all resonances are decomposed
dominant component: pion-pion rescattering
-> corresponds to random Levy walk
anomalous diffusion of pions in an expanding fireball

Detailed investigations:

Investigation of the role of cuts

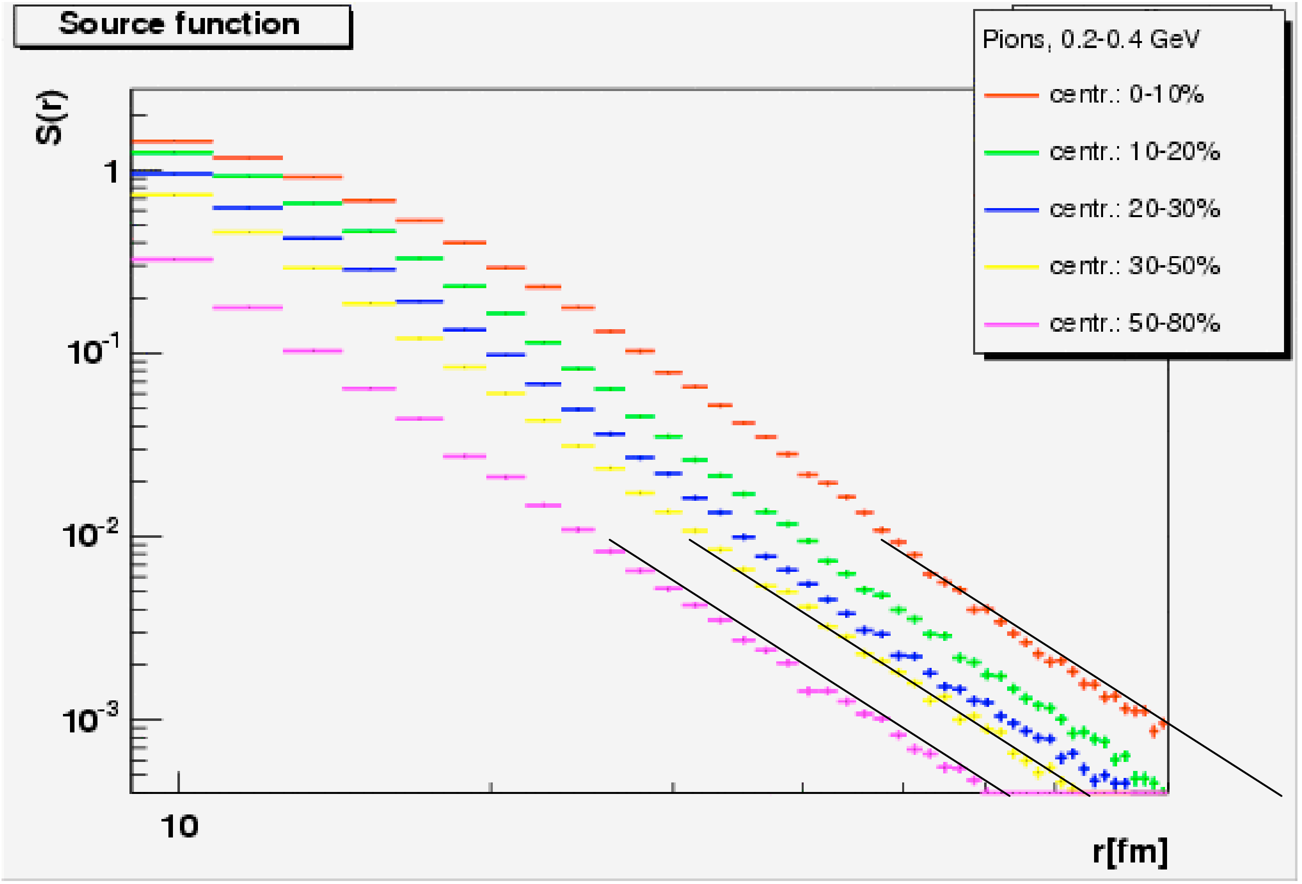
- -centrality
- -kT
- -PID

Is the effect particular or general,
valid for all cuts in nucl-ex/0605032 Fig. 3 (a-c,d-f)

Centrality	k_{T1} (MeV)	k_{T2} (MeV)	k_{T3} (MeV)
0-10 %	200-400	500-1000	-
10-20 %	same	s	-
20-30 %	s	s	-
30-50 %	s	s	-
50-80 %	s	s	-
0-20 %	200-360	360-480	480-600
40-80 %	same	same	same

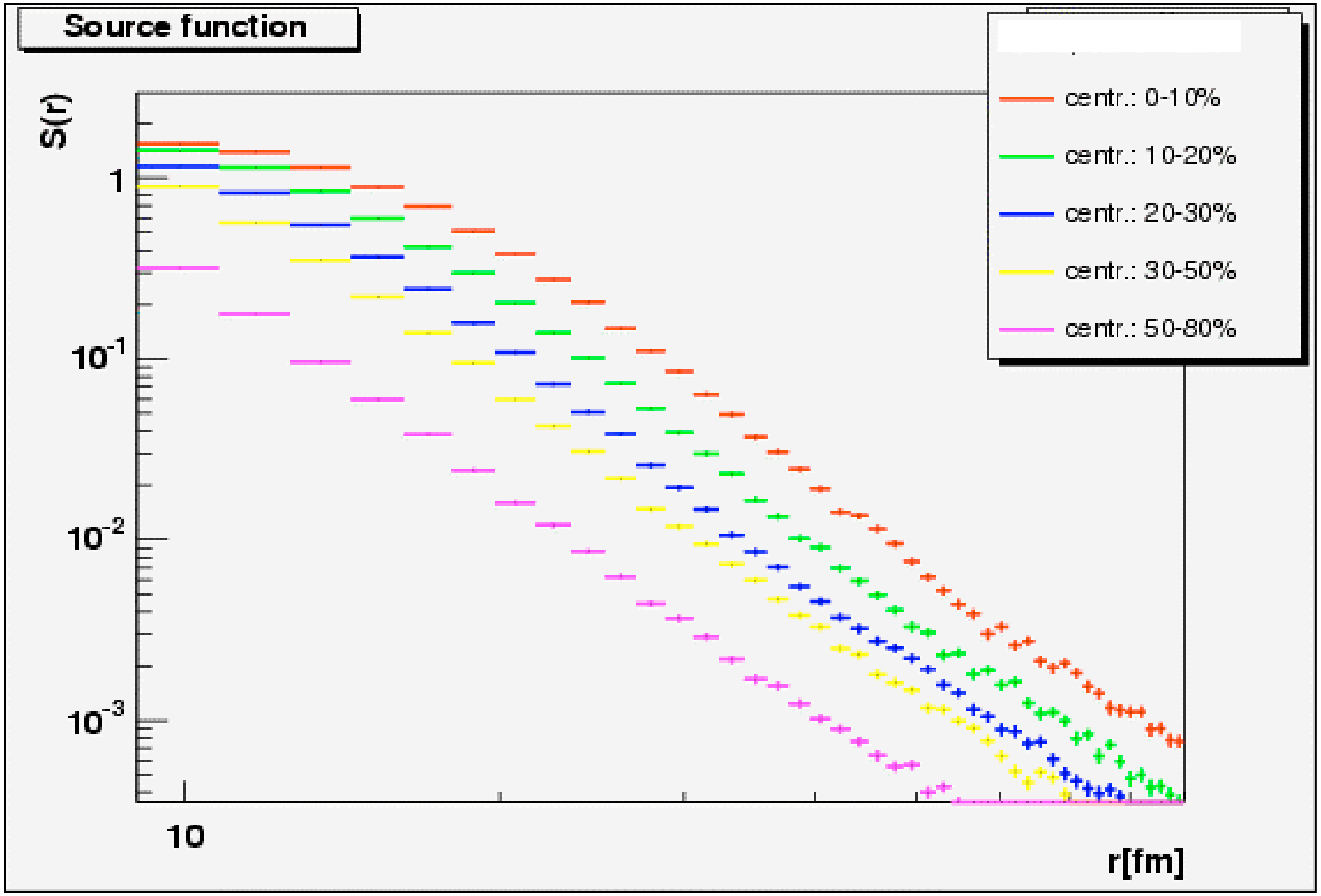
Centrality dependence

0.2 GeV < k_T < 0.4 GeV, pions



After a Gaussian start, a heavy tail in all centrality class exponent (log-log slope) weakly depends on centrality

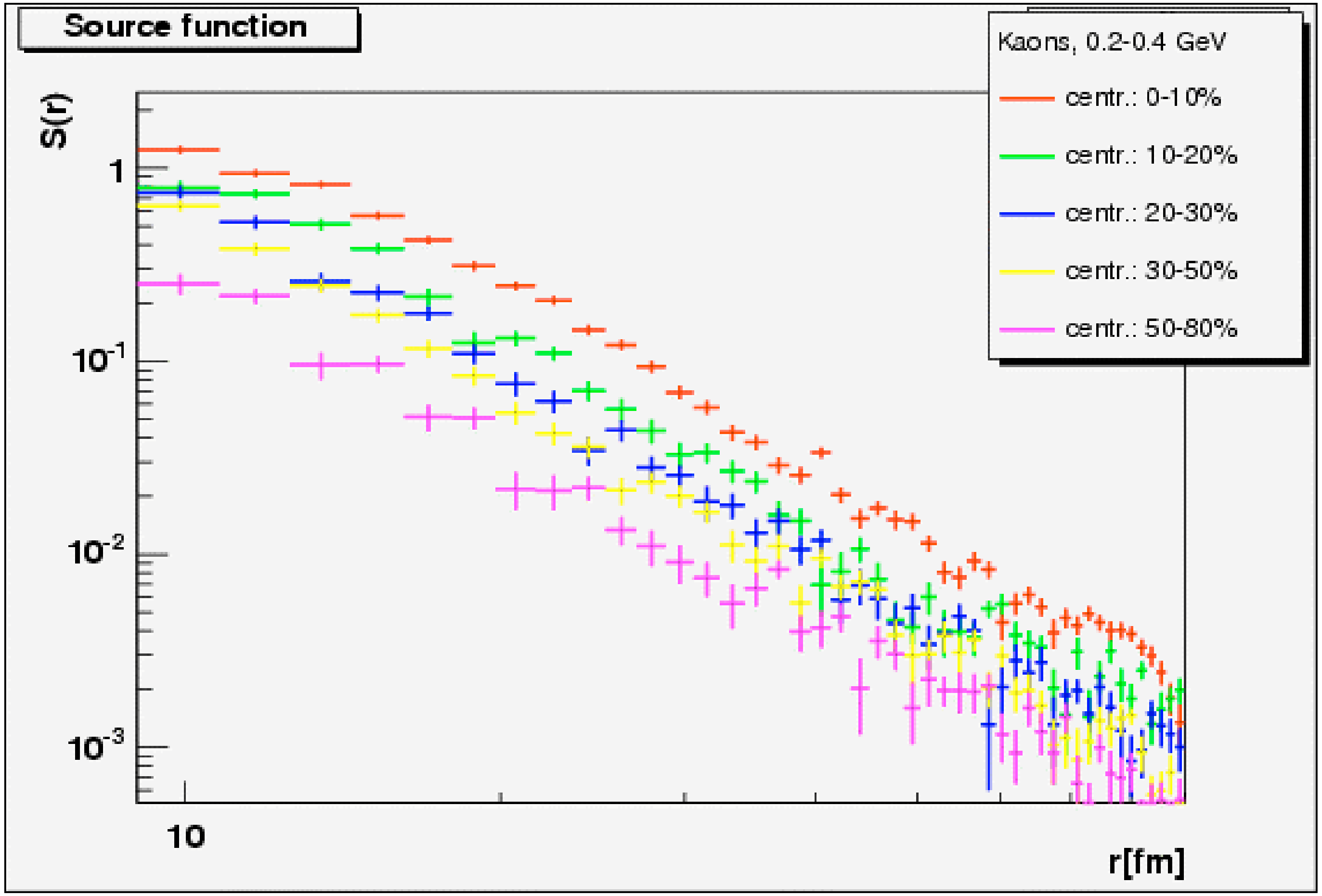
Centrality dependence,
0.5 GeV < k_T < 1.0 GeV, pions



Same as at low k_t: a heavy tail in all centrality class exponent (log-log slope) weakly depends on centrality

Centrality dependence

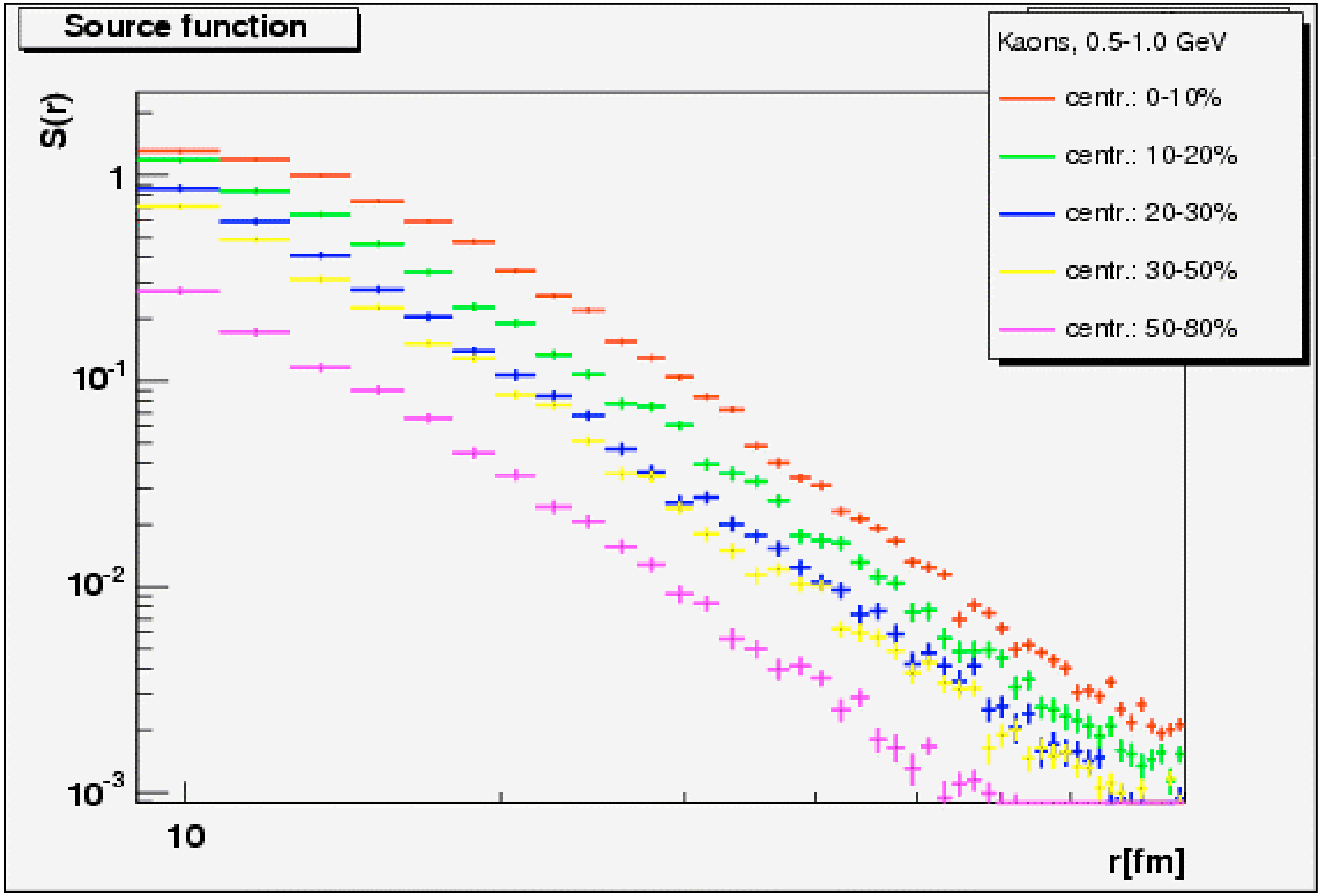
0.2 GeV < k_T < 0.4 GeV, kaons



The Gaussian width decreases with decreasing overlap,
a tail exists in all centrality class also for kaons
exponent (log-log slope) weakly depends on centrality

Centrality dependence

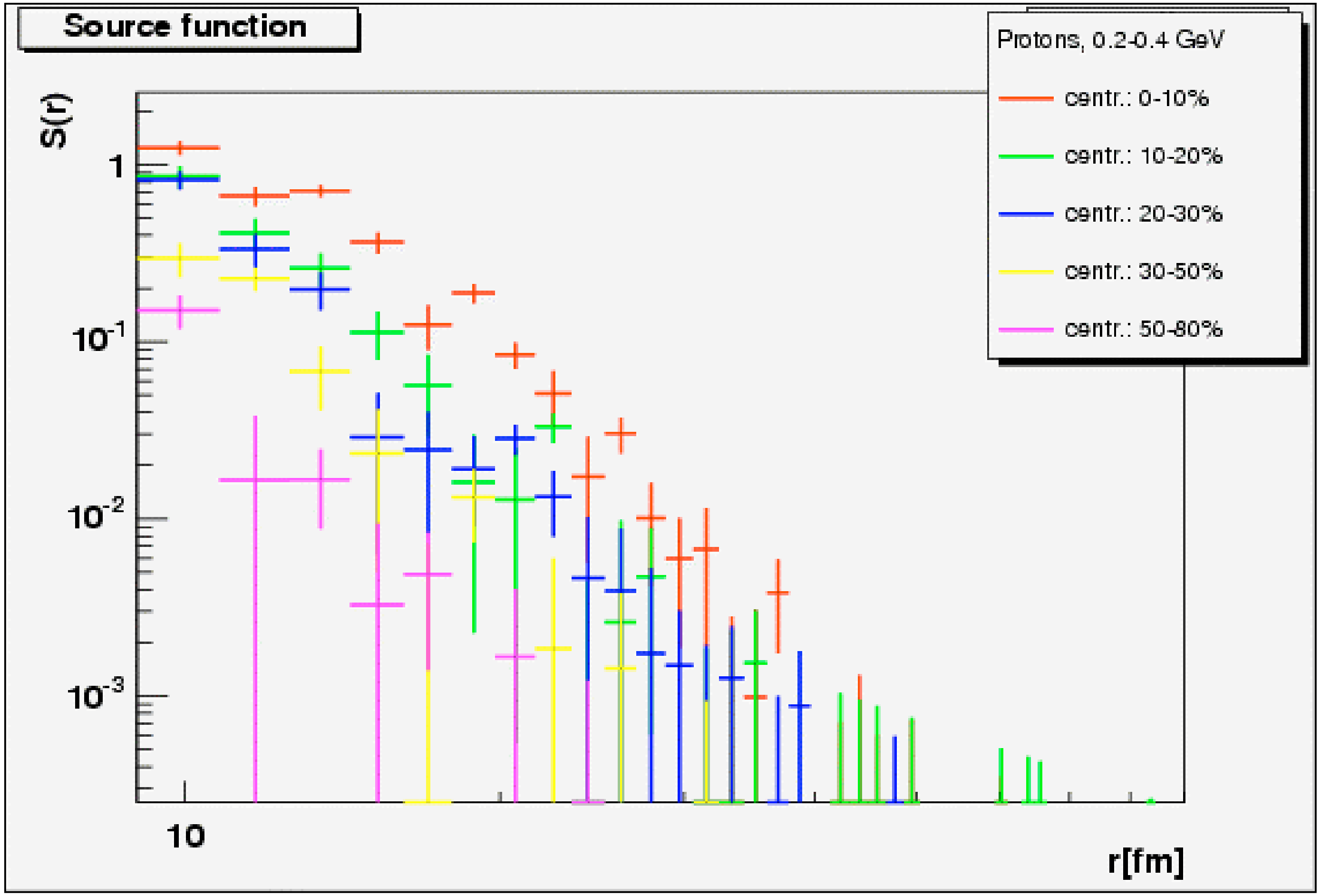
0.5 GeV < k_T < 1.0 GeV, kaons



The Gaussian width decreases with decreasing overlap,
a tail exists in all centrality class also for kaons at higher k_T
exponent (log-log slope) weakly depends on centrality

Centrality dependence

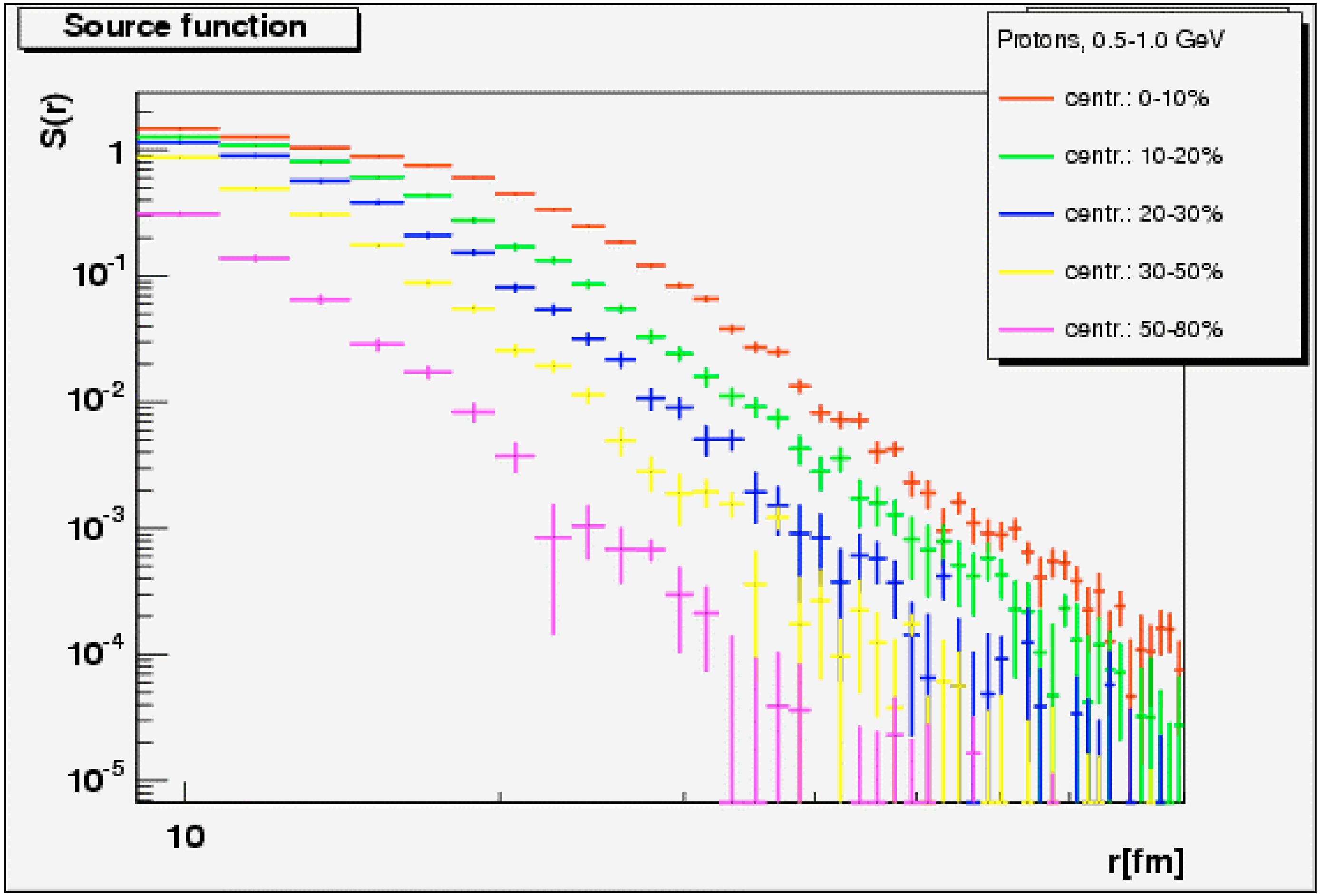
0.2 GeV < k_T < 0.4 GeV, protons



Problems with statistics in simulation for protons
exponent (log-log slope) weakly depends on centrality

Centrality dependence

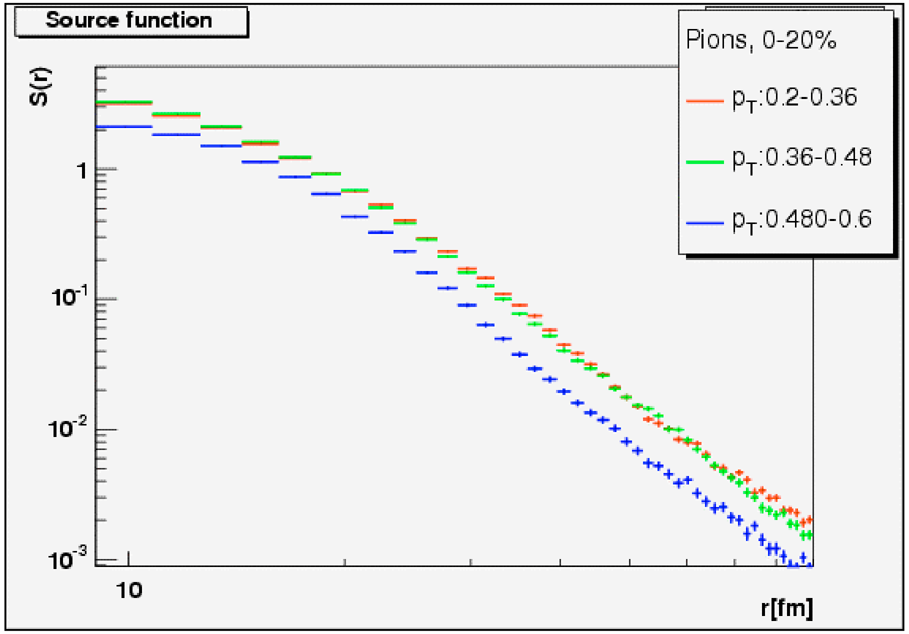
0.5 GeV < k_T < 1.0 GeV, protons

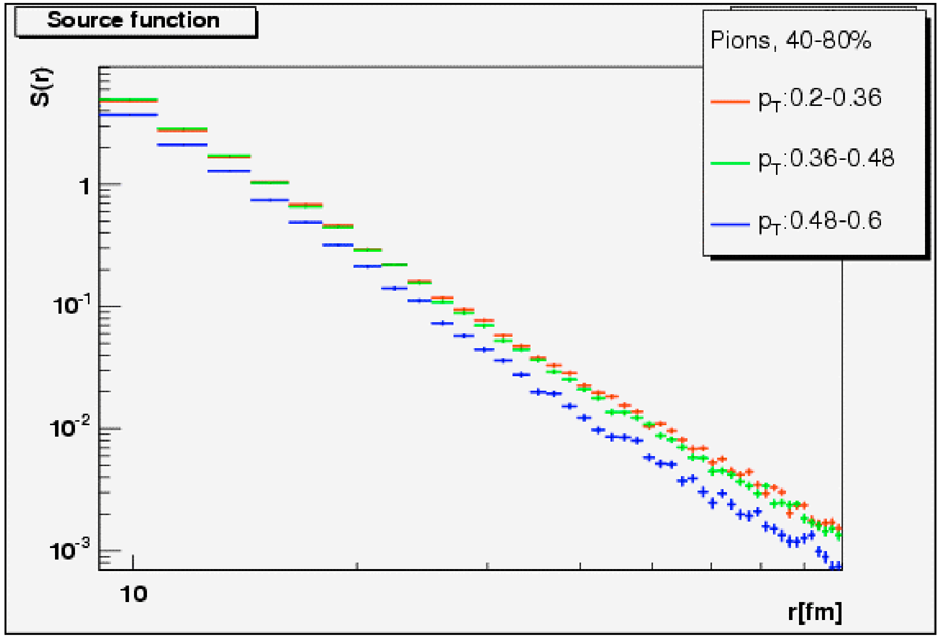


The Gaussian width decreases with decreasing overlap, a tail exists in all centrality class also for protons and weakly depends on centrality in the MC simulation

Momentum dependence

0-20 % and 40-80%, pions

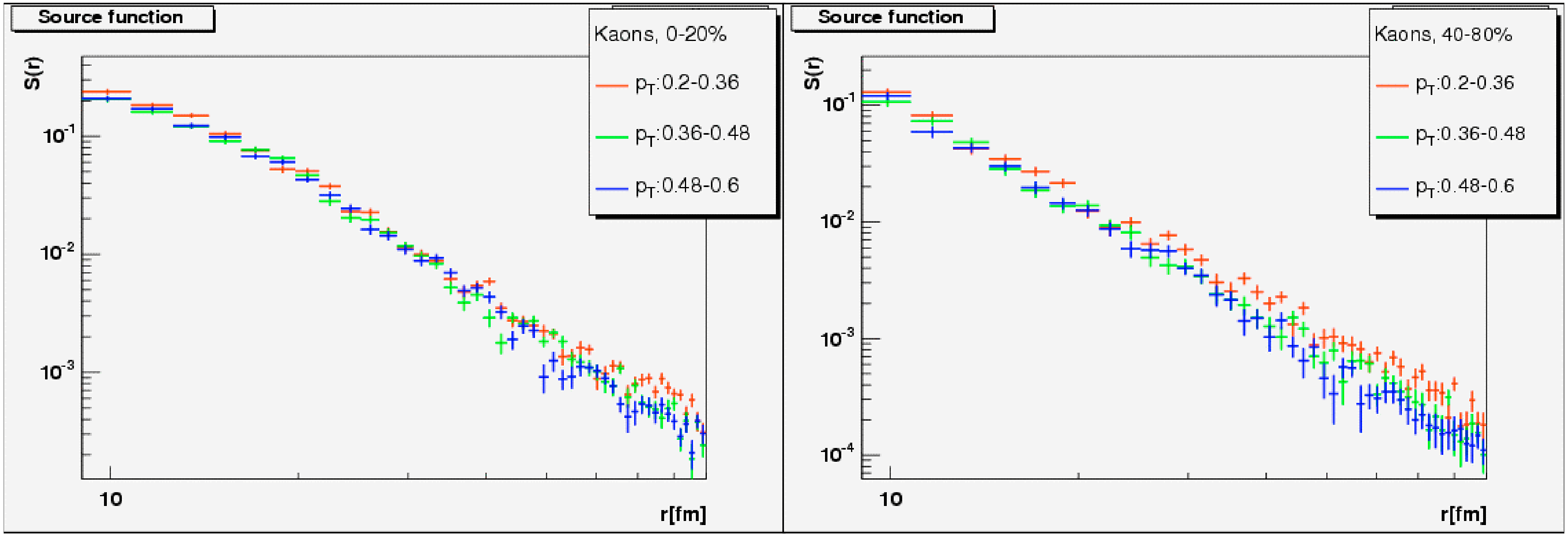




The Gaussian width decreases with decreasing overlap, a tail weakly depends on k_T in the MC simulation

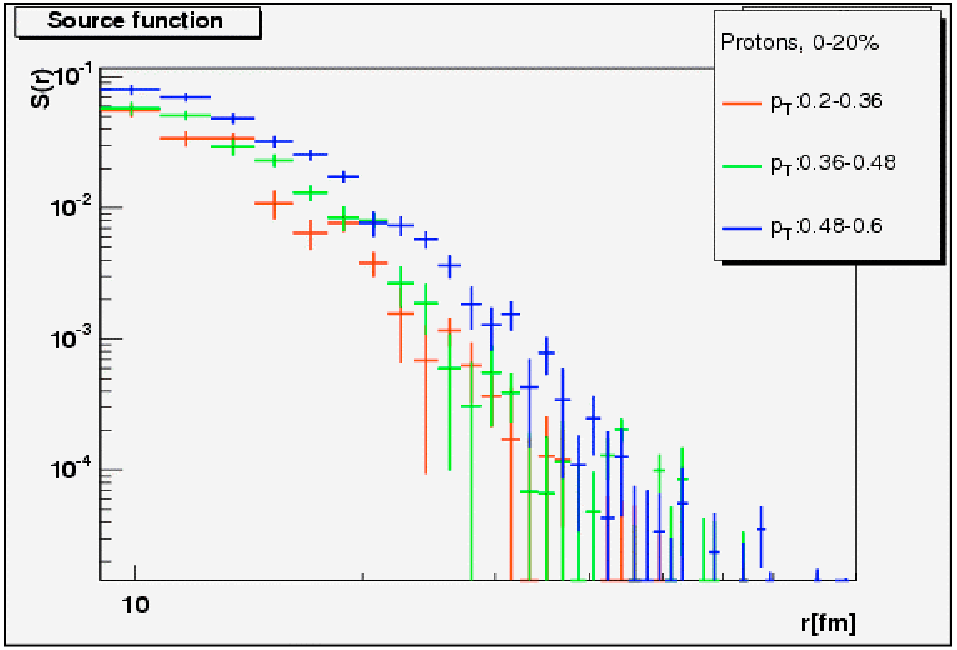
Momentum dependence

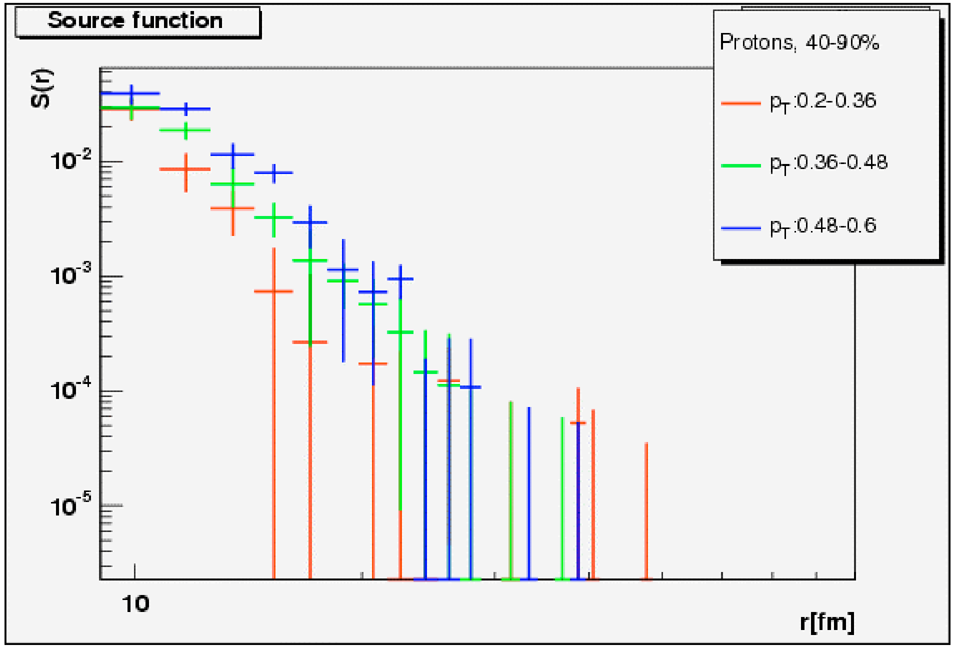
0-20 % and 40-80%, kaons



The Gaussian width decreases with decreasing overlap,
a tail weakly depends on k_T in the MC simulation
even for kaons

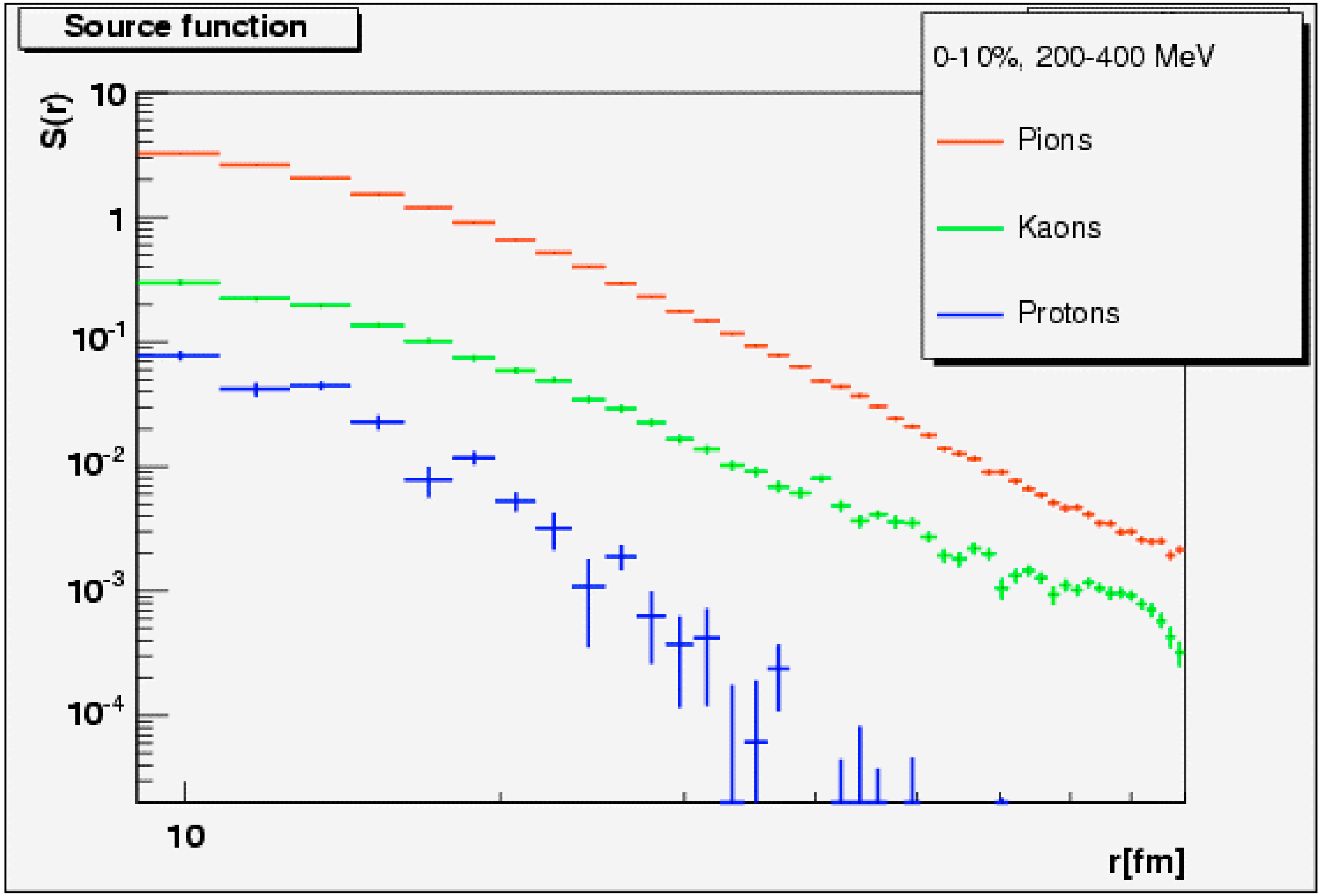
Momentum dependence
0-20 % and 40-80%, protons





The Gaussian width decreases with decreasing overlap,
a tail weakly depends on k_T in the MC simulation
even for protons - does it depend on anything at all??

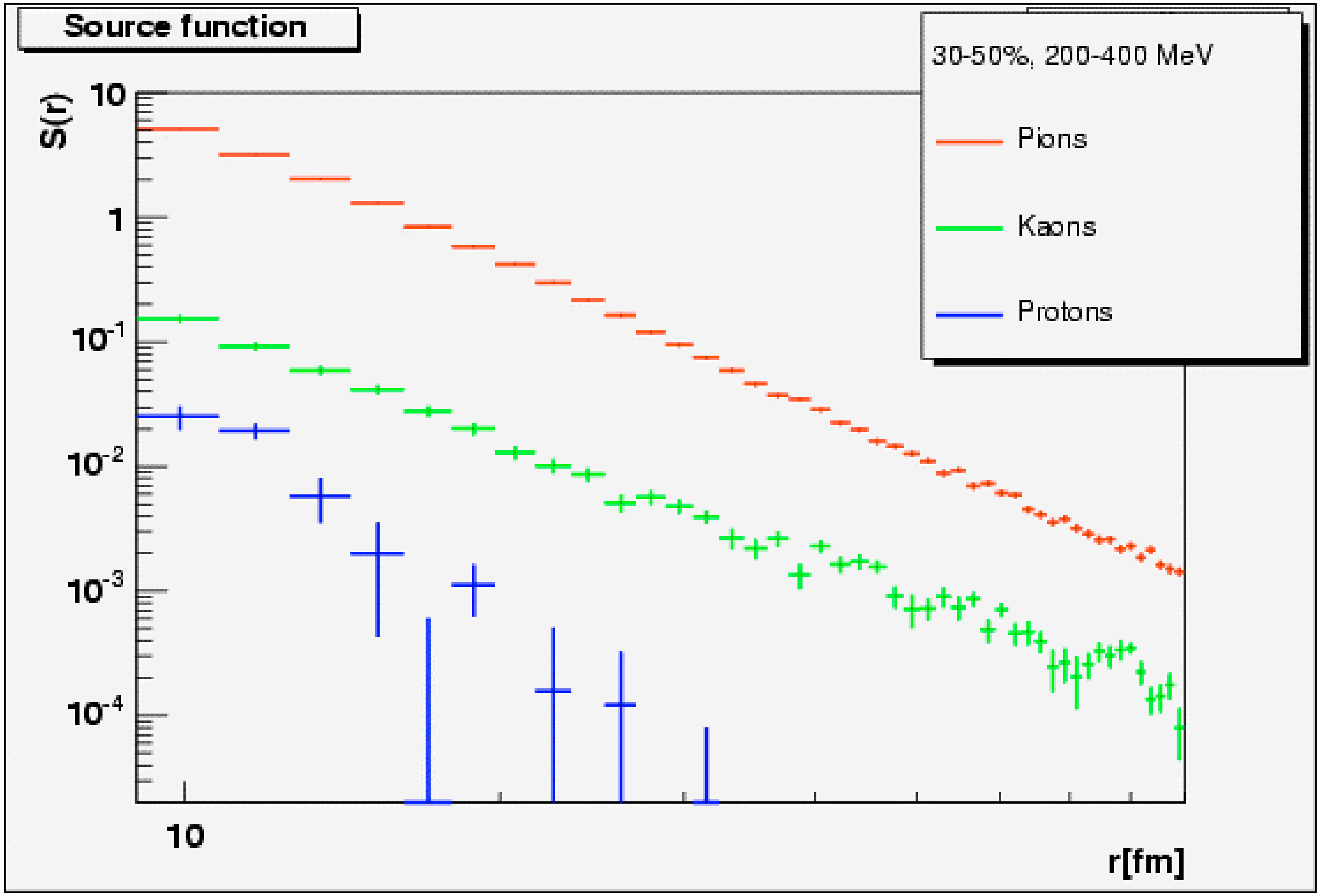
PID dependence
0-10 % and 200-400 MeV



The tail strongly depends on PID (particle type)
in the MC simulation
largest for kaons - that have the smallest cross sections

PID dependence

30-50 % and 200-400 MeV

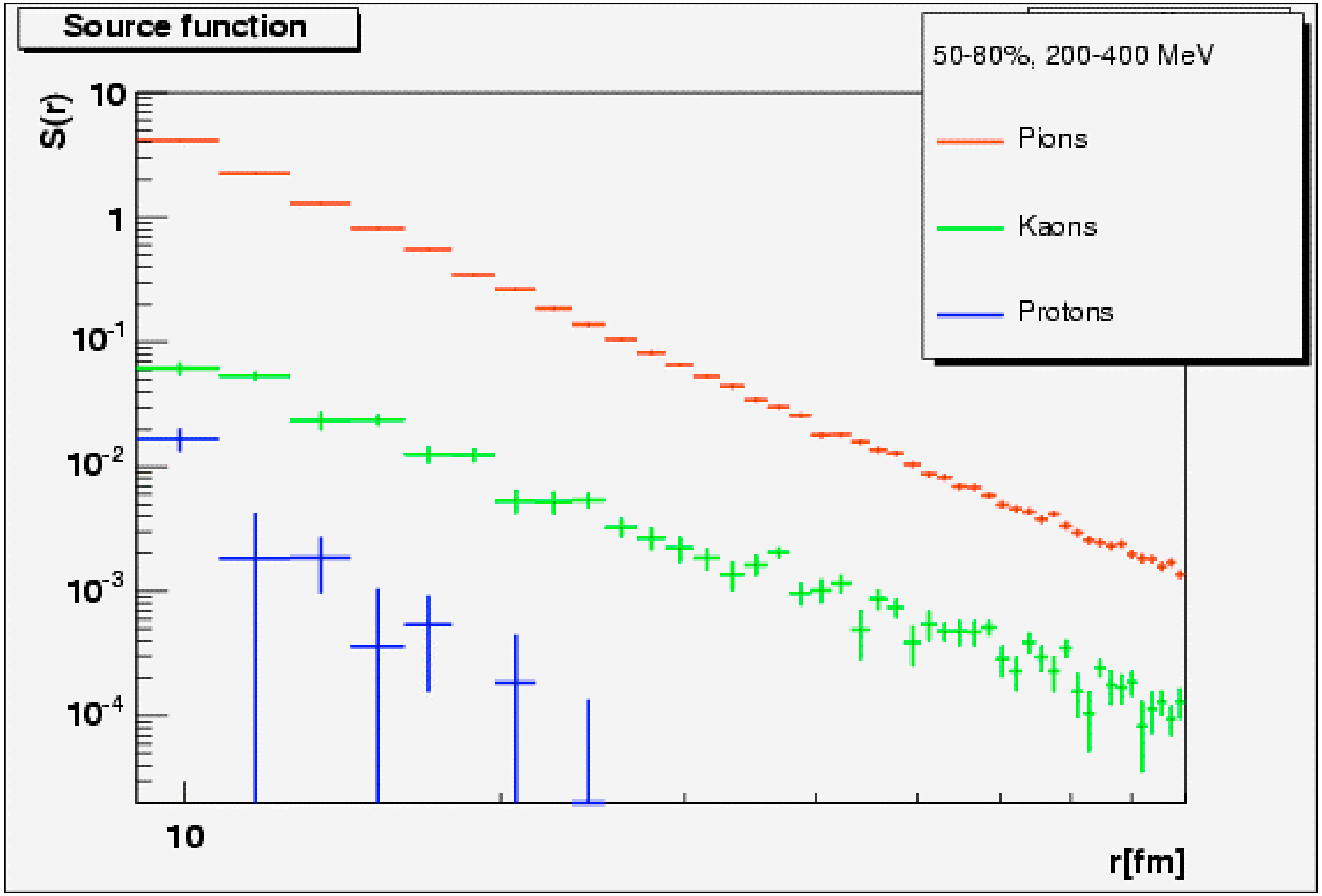


The tail strongly depends on PID

largest for kaons - that have the smallest cross sections

PID dependence

50-80 % and 200-400 MeV



The same, strong PID dependece

Conclusions, summary

In a hadronic rescattering MC simulation:

- a heavy tail exists
in all centrality, k_T range of PHENIX nucl-ex/0605032

- it is of a power-law type (linear on log-log plot)
$\alpha \sim 1.3$
- exponent weakly depends on centrality and k_T

- exponent strongly depends on PID
(particle cross section) in the MC simulation

Consequence:
a 2nd order QCD phase transition $\alpha \sim 0.5$
and rescattering lead to different Lévy exponents -
check PID dependence of α first!